Final Exam — EE 233  
Spring 2002

The test is closed book, with two sheets of 8.5 by 11 inch notes and standard calculators allowed. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses. Total of 150 points in 6 problems on 6 pages. A table of Laplace transform pairs are attached as page 7.

1. Determine the Thevenin equivalent of the circuit shown to the right. (20)

\[ V_{th} = V_{oc} = V_0 \]

\[ Z_C = \frac{1}{j\omega C} = \frac{-j}{(1000)(2 \times 10^{-4})} = -j5 \]

\[ Z_L = j\omega L = j(1000)(10^{-2}) = j/10 \]

\[ V_C = V_{oc} \left( \frac{-j5}{5-j5} \right) = V_{oc} \left( \frac{-j}{1-j} \right) = V_{oc} \left( \frac{1-j}{2} \right) \]

\[ 5\angle 0 = \frac{V_0}{5(1-j)} + \frac{V_0 - 10V_C}{j10} = \frac{V_0 (1+j)}{10} + \frac{V_0 \left[ 1-5(1-j) \right]}{j10} \]

\[ = \frac{V_0}{10} \left[ 1+j -j(1-5+5j) \right] = \frac{V_0}{10} \left[ 1+j + 4j + 5 \right] = V_0 \left( \frac{6+5j}{10} \right) \]

\[ V_{oc} = \frac{50}{6+5j} = 6.4 \angle 39.8 \]

\[ Z_{th} = \frac{V_{oc}}{I_{sc}} \quad V_0 = 0, \text{ so } V_C = 0 \]

\[ I_{sc} = 5\angle 0 \]

\[ Z_{th} = \frac{10}{6+5j} = 1.28 \angle 39.8 \]

\[ Z_{th} \]
2. In the circuit to the right, 
\( v_0 = 2 \text{ V} \) and \( i_L = 0.2 \text{ A} \) 
at \( t = 0^- \).

Draw the s-domain circuit valid for \( t > 0 \) and determine 
\( I_L(s) \). (25)

\[
L\left\{ e^{-2t} u(t) \right\} = \frac{1}{(s+2)^2}
\]

\[
2V = \frac{1}{2} F \quad \Rightarrow V_{OC} = \frac{2V}{s} \quad V_{OC} \pm \frac{2V}{s}
\]

\[
0.2A \downarrow \frac{1}{3} \text{ SH} \quad \Rightarrow sL = \frac{1}{3} s
\]

\[
I = \frac{2}{(s+2)^2} - \frac{2V}{s} + \frac{1}{s}
\]

\[
I_L = \frac{2 - (2V)(s+2)^2 + (s+2)^2}{(s+2)^2(2s + 2 + 5s^2)}
\]

\[
\text{Test w/ IVT} \quad \lim_{s \to 0} sI_L = \lim_{s \to 0} \frac{s^4 + \ldots}{s^5} = \frac{1}{5} = 0.2 \checkmark
\]
3. The output of a circuit in the s-domain is:

\[ V(s) = \frac{10s}{(s+3)^2} + \frac{s^2}{(s^2 + 4s + 3)} \]

Find \( v(t) \). (25)

\[ \frac{10s}{(s+3)^2} = \frac{K_1}{(s+3)^2} + \frac{K_2}{s+3} \]

\[ K_1 = \frac{10s}{(s+3)^2} \bigg|_{s=-3} = -30 \]

\[ K_2 = \frac{d}{ds} \left[ 10s \right]_{s=-3} = 10 \]

\[ \frac{s^2}{s^2 + 4s + 5} = 1 - \frac{4s + 5}{s^2 + 4s + 5} \]

\[ = 1 + \frac{K}{s+2-j} + \frac{K^*}{s+2+j} \]

\[ K = \frac{-4s - 5}{s+2+j} \bigg|_{s=-2+j} = \frac{8 - 4j - 5}{-2 + j + 2 + j} \]

\[ = \frac{3 - 4j}{2j} = -2 - \frac{3}{2}j = \frac{5}{2} \angle 143.1^{\circ} \]

Check: For \( s = -2 \)

\[ \frac{-30}{(s+3)^2} + \frac{10}{s+3} \]

\[ = \frac{-2 - \frac{3}{2}j}{-2 + j + 2 + j} + \frac{-2 + \frac{3}{2}j}{-2 + j + 2 + j} \]

\[ = \frac{2 + \frac{3}{2}j}{-j} + \frac{-2 + \frac{3}{2}j}{j} \]

\[ = 3 \]

\[ \frac{5}{2} \angle 143.1^{\circ} + \frac{5}{2} \angle 143.1^{\circ} \]

\[ = \frac{5e^{-2t} \cos(t - 143.1^\circ)}{s^2 + 4s + 5} \]

\[ = \frac{-4s - 5}{s^2 + 4s + 5} = \frac{8 - 5}{4 - 8 + 5} = 3 \]

\[ \mathcal{L}^{-1} \left\{ \frac{-30}{(s+3)^2} + \frac{10}{s+3} \right\} = \left[ -30e^{-3t} + 10e^{-3t} \right] u(t) \]

\[ \mathcal{L}^{-1} \left\{ \frac{5}{2} \angle 143.1^{\circ} + \frac{5}{2} \angle 143.1^{\circ} \right\} = 5e^{-2t} \cos(t - 143.1^\circ) \]

\[ \mathcal{L}^{-1} \left\{ 1 \right\} = \delta(t) \]

\[ v(t) = \delta(t) + \left[ 10e^{-3t} - 30e^{-3t} + 5e^{-2t} \cos(t - 143.1^\circ) \right] u(t) \]

\[ 143.1^\circ = 2.5 \text{ rad} \]
4. For the transfer function

\[ H(s) = \frac{s(s + 10^3)}{(s^2 + 2000s + 10^8)} \]

Sketch the asymptotic Bode plot (gain in dB and phase in degrees versus \( \log_{10} \omega \)). (25)

- On both plots, indicate the slopes of the asymptotic behavior within each frequency range.
- At the corner frequencies of the amplitude, calculate the amplitude and phase for the asymptotic approximation.
- On the amplitude plot, also indicate the actual gain at each corner frequency.

\[ s^2 + 2000s + 10^8 = s^2 + \frac{\omega_o}{Q} s + \omega_o^2 \implies \omega_o = 10^4, \; Q = 5 \]

At low frequencies: \( H(s) \approx \frac{10^5 s}{10^8} \implies H(j\omega) \approx \frac{j\omega}{10^3} \]

\[ |\frac{j\omega}{10^3}| = \left| \frac{j10^5}{\omega} \right| = 10 \]

Zeros at \( \omega = 0, 10^5 \)

2 poles at \( \omega = 10^4 \)

For \( 10^4 < \omega < 10^5 \)

\[ H(j\omega) \approx \frac{j\omega (10^5)}{-\omega^2} = -\frac{j10^5}{\omega} \]

At high frequencies (\( \omega > 10^5 \)):

\[ |H(j\omega)| \approx 34 \text{dB} \]

\[ 20 \log_{10} Q = 20 \log_{10} 5 = 14 \text{dB} \]

Note that we did not discuss how to plot Bode angle plots this quarter, so I would not ask for this to be drawn on the final. However, you should be able to calculate phase shift at any given frequency.
5. A filter is constructed using the circuit at the right.

Find the transfer function \( H(s) \) in terms of \( R_1 \), \( R_2 \), \( C_1 \), and \( C_2 \).

What type of filter is it? (20)

\[
\begin{align*}
V_+ &= V_- = V_a, & V_+^2 &= V_-^2 = V_o \\
\text{At node } V_+^1: & (V_i - V_a)SC_1 = \frac{(V_a - V_o)}{R_1} \quad [I] \\
\text{At node } V_+^2: & (V_a - V_o)SC_2 = \frac{V_o}{R_2} \Rightarrow V_a = V_o \left(1 + \frac{1}{SR_2C_2}\right) \\
\text{in } [I]: & V_i - \frac{V_o(1 + SR_2C_2)}{SR_2C_2} = \frac{1}{SR_1C_1} \left[\frac{V_o}{SR_2C_2}\right] \\
V_i' = V_o \left[\frac{1 + SR_1C_1(1 + SR_2C_2)}{SR_1R_2C_1C_2} \right] \\
\frac{V_o}{V_i'} = \frac{SR_1R_2C_1C_2}{SR_1R_2C_1C_2 + SR_1C_1 + 1} = \frac{S^2}{S^2 + \frac{1}{R_2C_2}S + \frac{1}{R_1R_2C_1C_2}}
\end{align*}
\]
6. Given these filter circuits with associated transfer functions:

\[
\begin{align*}
\frac{10\text{nf}}{s^2} & + \frac{10\text{nf}}{160\text{ks}} \quad & \frac{s^2}{s^2 + 2s/(R_2C) + 1/(R_1R_2C^2)} & + \frac{-1/(R_1C)}{s + 1/(R_2C)} \\
\frac{-sR_2/R_1}{s + 1/(R_1C)} & + \frac{2.5\text{Ms}}{s^2 + 2s/(R_1C) + 1/(R_1R_2R_3C^2)} & \frac{-s/(R_1C)}{s^2 + 2s/(R_3C) + (R_1 + R_2)/(R_1R_2R_3C^2)} \\
\frac{1/(R^2C_1C_2)}{s^2 + 2s/(RC_1) + 1/(R^2C_1C_2)} & &
\end{align*}
\]

Using the above active filter blocks, design a Butterworth highpass filter with cutoff frequency of \( \omega_c = 2000 \text{ rad/s} \) and a pass-band gain of magnitude 34dB. If the gain must be below 0dB for frequencies below 500 rad/s, what is the minimum filter order? Specify all component values, using 10nF capacitors whenever possible without increasing the part count. (30)

Butterworth: \( \omega_c, \omega_s = \frac{500}{2000} = \frac{1}{4} \) \( \log_{10} \left( \frac{1}{4} \right) = -0.6 \) (60% 1st decade)

Since gain must drop by 34dB, required order \( \frac{34\text{dB}}{0.6\text{dB/decade}} = 2.83 \leq n \)

Thus, a 3rd order HPF is required. More accurate Butterworth equation gives \( n \geq \log_{10} \left( \frac{10^{34/60} - 1}{2\log_{10}(4)} \right) = 2.82 \).

Circuit is illustrated above:

For 2nd order HPF \( \omega_c = \sqrt{R_1R_2C^2} = 2000 = \omega_c \), so \( R_1R_2C^2 = 2.5 \times 10^{-7} \)

For 3rd order Butterworth, \( Q \) of 2nd order stage is 1 (no drop, 3dB dip from 1st order)

\[ \frac{\omega_0}{Q} = \frac{2000}{1} = \frac{2}{R_2C}. \] This can both be satisfied with \( C = 10\text{nf} \)

\[ R_2 = \frac{2}{2000C} = \frac{2}{2 \times 10^3 \times (10^{-8})} = 10^5 \Omega = 100 \text{ k}\Omega, \] \[ R_1 = \frac{2.5 \times 10^{-7}}{(10^5)(10^{-8})^2} = \frac{2.5 \times 10^4 \Omega}{25 \text{ k}\Omega} \]

For 1st order HPF \( \omega_s = 1/R_1C = 2000, \) so \( R_1C = 5 \times 10^{-4}, \) using \( C = 10\text{nf} \)

\[ R_1 = \frac{5 \times 10^{-9}}{10^{-8}} = 5 \times 10^4 = 50 \text{k}\Omega \] Pass band gain of 2nd order HPF = 1, so 1st order needs gain of 34dB or \( 50^6 = \frac{R_2}{R_1}, \) so \( R_2 = 2.5 \text{MS} \)