2. A filter is constructed using the circuit at the right. Find the transfer function \( H(s) \) in terms of \( R_1, R_2, C_1, C_2, \) and \( C_3 \). What kind of filter is it? (25)
3. Design the $s$-domain transfer function for a low-pass filter with a corner frequency of $10^4$ rad/s, a pass-band gain of 50, a gain at the corner frequency of 37 dB, and a gain less than 0.5 at $8 \times 10^4$ rad/s. (25)
3. (a) Find the transfer function $H(s) = \frac{V_o(s)}{V_i(s)}$ for the circuit to the right. Assume no stored energy in circuit at $t = 0$. (15)

\[
\begin{align*}
V_o(s) &= H(s) \frac{Z_{R||C_2}}{Z_{C_1} + Z_{R||C_2}} = \frac{R}{SRC_1 + SRC_2} = \frac{SRC_1}{SRC_2 + 1} \\
&= \frac{0.02s}{0.025s + 1} = \frac{0.8s}{s + 40}
\end{align*}
\]

(b) Calculate the impulse response for this circuit. (10)

\[
\begin{align*}
h(t) &= L^{-1}\{H(s)\} = L^{-1}\{0.8 - \frac{32}{s + 40}\} \\
&= 0.8\delta(t) - 32e^{-40t}u(t)
\end{align*}
\]

(c) If $v_i(t) = u(t) - u(t - 50)$, calculate and sketch $v_o(t)$ versus time (seconds). (15)

Option 1: $V_o(s) = V_i(s)H(s) = \left(\frac{1}{s} - \frac{e^{-50s}}{s}\right)\left(\frac{0.8s}{s + 40}\right) = \frac{0.8(1-e^{-50s})}{s+40}$

\[
\begin{align*}
v_o(t) &= L^{-1}\{0.8(1-e^{-50s})\} = 0.8e^{-40t}u(t) - 0.8e^{-40(t-50)}u(t-50)
\end{align*}
\]

Option 2: $v_o(t) = v_i(t) \ast h(t)$

\[
\begin{align*}
= \int_{0}^{\infty} v_i(\lambda - t)h(\lambda) d\lambda \\
= \int_{0}^{\infty} v_i(\lambda)h(\lambda) d\lambda = \frac{32}{40}e^{-40} \left\{ 0.8 \int_{-t-50}^{t-50} 0.8\delta(\lambda - 32e^{-40t}) d\lambda + 0.8 \int_{t-50}^{t} e^{-40t} d\lambda \right\}
\end{align*}
\]

End Of Exam