Homework 1 Solutions

1 J&B P2.5(a)

\[ R = \frac{\rho L}{A} = \frac{1.66 \, \mu\Omega \cdot 2\sqrt{2}}{5 \cdot 10^{-8} \, \text{cm}^2} = 93.9 \, \Omega \]

2 J&B P2.6

Intrinsic carrier concentration is given by

\[ n_i^2 = BT^3 e^{\frac{E_G}{kT}} \]

for germanium, \( E_G = 0.66 \, eV \), \( B = 2.31 \times 10^{30} \, K^{-3} \, \text{cm}^{-6} \)

(a) At 77 K, \( n_i^2 = 2.63 \times 10^{-4} \, \text{cm}^{-3} \)

(b) At 300 K, \( n_i^2 = 2.27 \times 10^{13} \, \text{cm}^{-3} \)

(a) At 500 K, \( n_i^2 = 8.04 \times 10^{15} \, \text{cm}^{-3} \)

3 J&B P2.16

Maximum hole current density is

\[ j_p = q \mu_p = 1.60 \times 10^{-19} \, \text{C} \cdot 10^{19} \, \text{cm}^{-3} \cdot 10^7 \, \text{cm/s} = 1.60 \times 10^7 \, \text{A/cm}^2 \]

The maximum current is

\[ I = j_p A = 1.60 \times 10^7 \, \text{A/cm}^2 \cdot 2.5 \times 10^{-7} \, \text{cm}^2 = 4 \, A \]

4 J&B P2.22

In has 3 outer electrons, P has 5 outer electrons. Ge has 4 outer electrons.

(a) Germanium has one more electron than indium. If a germanium atom replaces an indium atom, it can donate the extra electron for conduction. Thus it behaves as a donor impurity.

(b) Germanium has one less electron than phosphorus. If a germanium atom replaces a phosphorus atom, it can accept an electron from neighboring bonds, creating a free hole. Thus it behaves as an acceptor impurity.

5 J&B P2.27

\[ N_A = 6 \times 10^{18} \, \text{boron atoms/cm}^3 \]

(a) This is p-type silicon; holes are the majority carrier; electrons are the minority carrier.
For this problem, we need the intrinsic carrier concentration at two different temperatures. The more precise formula for $n_i$ in Eq. (2.1) will be used to compute this:

$$n_i = \sqrt{BT^3 e^{-\frac{E_G}{kT}}}$$

$$n_i(300K) = 1.0 \times 10^{10} \text{ cm}^{-3}$$

$$n_i(200K) = 1.1 \times 10^5 \text{ cm}^{-3}$$

For both 300K and 200K, the majority carrier hole concentration is just equal to

$$N_A = 6 \times 10^{18} \text{ cm}^{-3}$$

However, the minority carrier electron concentrations depends upon $n_i(T)$:

$$n_{300K} = \frac{(n_i(300K))^2}{N_A} = 16.7 \text{ cm}^{-3}$$

$$n_{200K} = \frac{(n_i(200K))^2}{N_A} = 2.02 \times 10^{-9} \text{ cm}^{-3}$$

6 J&B P2.32

$$N_D > N_A$$

Then

$$n = \frac{(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2} = 1.62 \times 10^{17} \text{ cm}^{-3}$$

$$p = \frac{n_i^2}{n} = 6.17 \times 10^{16} \text{ cm}^{-3}$$

We can see that

$$n = 1.62 \times 10^{17} \text{ cm}^{-3} \neq N_D - N_A = 1 \times 10^{17} \text{ cm}^{-3}$$

This is because

$$N_D - N_A \gg n_i$$

does not hold in this case ($N_D - N_A = n_i = 1 \times 10^{17} \text{ cm}^{-3}$)

7 J&B P2.35

Indium is an acceptor impurity. The material is p-type.

At 300 K, the intrinsic carrier concentration for silicon is $n_i = 1.0 \times 10^{10} \text{ cm}^{-3}$. 
Acceptor doping concentration is \( N_A = 8 \times 10^{19} \text{ cm}^{-3} \). Thus \( N_A \gg n_i \), and we have

\[
p = N_A = 8 \times 10^{19} \text{ cm}^{-3}
\]

\[
n = \frac{n_i^2}{p} = 1.25 \text{ cm}^{-3}
\]

Using total impurity concentration of \( N_T = N_A = 8 \times 10^{19} \text{ cm}^{-3} \), we can extract the electron and hole mobilities by either reading them out from Figure 2.8, or plugging in the value of \( N_T \) in the approximated mobility equations:

\[
\mu_n = 96 \text{ cm}^2/(V \cdot \text{s})
\]
\[
\mu_p = 50 \text{ cm}^2/(V \cdot \text{s})
\]

Conductivity

\[
\sigma = q(n \mu_n + p \mu_p)
\]

Since

\[
p \gg n
\]

The electron part can be neglected, and thus

\[
\sigma = q p \mu_p = 640 (\Omega \cdot \text{cm})^{-1}
\]

Resistivity

\[
\rho = \frac{1}{\sigma} = 1.56 \times 10^{-3} \Omega \cdot \text{cm}
\]

8 J&B P2.50

Diffusion current density of holes is

\[
j_p^{\text{diff}} = -q D_p \frac{dp}{dx}
\]

The concentration profile for holes is

\[
p(x) = 10^5 + 10^{19} e \left(-\frac{x}{L_p}\right) \text{ cm}^{-3}
\]

Then

\[
j_p^{\text{diff}} = 10^{19} q \frac{D_p}{L_p} e \left(-\frac{x}{L_p}\right) \text{ cm}^{-3} = 1.2 \times 10^5 e \left(-\frac{x}{2 \times 10^{-4} \text{ cm}}\right) A/\text{cm}^2
\]

At \( x = 0 \), diffusion current density is
\[ j_{p}^{\text{diff}}(x = 0) = 1.2 \times 10^5 \text{ A/cm}^2 \]

Diffusion current is

\[ I_{p}^{\text{diff}}(x = 0) = j_{p}^{\text{diff}}(x = 0) \cdot A = 1.2 \times 10^{-2} \text{ A} \]

**7 J&B P2.52**

Note that the positive \( x \) direction is to the right, and the direction of the electric field is to the left, thus \( E = -20 \text{ V/cm} \). (The signs of \( E, \frac{dn}{dx}, \frac{dp}{dx}, j \) are very important in this exercise!) The concentration profiles are linear, thus the gradients of concentrations are

\[ \frac{dp}{dx} = \frac{\Delta p}{\Delta x} = -5 \times 10^{19} \text{ cm}^{-4} \]
\[ \frac{dn}{dx} = \frac{\Delta n}{\Delta x} = -5 \times 10^{19} \text{ cm}^{-4} \]

Mobilities of electrons and holes are \( \mu_n = 350 \text{ cm}^2/(\text{V} \cdot \text{s}) \), \( \mu_p = 150 \text{ cm}^2/(\text{V} \cdot \text{s}) \). From Einstein’s relationship

\[ \frac{D}{\mu} = \frac{kT}{q} = V_T \]

The diffusivities of electrons and holes are (assuming room temperature, \( V_T = 25.9 \text{ mV} \))

\[ D_n = \mu_n V_T = 9.1 \text{ cm}^2/\text{s} \]
\[ D_p = \mu_p V_T = 3.9 \text{ cm}^2/\text{s} \]

At \( x = 0 \), \( n = 10^{16} \text{ cm}^{-3} \), \( p = 1.01 \times 10^{18} \text{ cm}^{-3} \)

Electron diffusion current density:

\[ j_n^{\text{diff}} = -qD_n \frac{dn}{dx} = -72 \text{ A/cm}^2 \]

Hole diffusion current density:

\[ j_p^{\text{diff}} = -qD_p \frac{dp}{dx} = 31 \text{ A/cm}^2 \]

Electron drift current density:

\[ j_n^{\text{drift}} = qn\mu_n E = -11.2 \text{ A/cm}^2 \]

Hole drift current density:
\[ j_p^{drift} = q \mu_p E = -4.85 \times 10^2 \, \text{A/cm}^2 \]

Total current density:
\[ j_{tot} = j_n^{diff} + j_p^{diff} + j_n^{drift} + j_p^{drift} = -5.4 \times 10^2 \, \text{A/cm}^2 \]

Since the profiles are linear, at \( x = 1 \mu m \) (mid-point)
\[ n = \frac{(10^{16} + 10^4)}{2} = 5 \times 10^{15} \, \text{cm}^{-3} \]
\[ p = \frac{(1.01 \times 10^{18} + 10^{18})}{2} = 1.005 \times 10^{18} \, \text{cm}^{-3} \]

Electron and hole diffusion currents are the same as derived above:
\[ j_n^{diff} = -q D_n \frac{dn}{dx} = -72 \, \text{A/cm}^2 \]
\[ j_p^{diff} = -q D_p \frac{dp}{dx} = 31 \, \text{A/cm}^2 \]

Electron drift current density:
\[ j_n^{drift} = q n \mu_n E = -5.6 \, \text{A/cm}^2 \]

Hole drift current density:
\[ j_p^{drift} = q p \mu_p E = -4.8 \times 10^2 \, \text{A/cm}^2 \]