MOSFET Current versus Voltage with Linearized Depletion Charge

The inversion charge is given by:

$$Q_I'(y) = -\left[V_{GB} - V_{FB} - (V_{CB}(y) - 2\phi_F)\right] C_{ox}' - Q_B'(y). \tag{1}$$

Rather than using

$$Q_B'(y) = \mp q N_B x_{d\,\max}(y) = \mp\sqrt{2K_s\epsilon_0 q N_B |V_{CB}(y) - 2\phi_F|}, \tag{2}$$

we can instead approximate $Q_B'(y)$ by its first order Taylor series around $V_{CB} = V_{SB}$:

$$\begin{aligned} Q_B'(y) &\cong Q_B'(0) + V_{CS}\left(dQ_B'(y)/dV_{CB}\right)_{V_{SB}} \\ &= \mp\sqrt{2K_s\epsilon_0 q N_B |V_{SB}(y) - 2\phi_F|} - V_{CS}\sqrt{\frac{K_s\epsilon_0 q N_B}{2|V_{SB}(y) - 2\phi_F|}} \end{aligned} \tag{3}$$

Thus,

$$Q_I'(y) = -\left[V_{GS} - V_{th} - (1+\alpha)V_{CS}(y)\right] C_{ox}', \tag{4}$$

where

$$\alpha = \frac{1}{C_{ox}'}\sqrt{\frac{K_s\epsilon_0 q N_B}{2|V_{SB} - 2\phi_F|}} = \frac{C_d'}{C_{ox}'} \tag{5}$$

Integrating from $y = 0$ to L and $V_{CS} = 0$ to V_{DS},

$$I_{DS} = \pm\frac{W}{L}\mu' C_{ox}'\left[V_{GS} - V_{th} - (1+\alpha)\frac{V_{DS}}{2}\right] V_{DS} \tag{6}$$

where V_{th} includes the effects of ϕ_F, ϕ_{MS}, oxide charges and $Q_B'(y = 0)$.

For a typical device, the approximate expression (6) works well in capturing the effect of changing depletion charge. For large V_{DS}, a slightly smaller value of α based on expanding V_{CB} around a value between V_{DB} and V_{SB} may be better.

Equation (6) is only valid when an inversion layer exists all the way across the channel from the source to the drain. As V_{DS} increases, the voltage between the gate and the channel near the drain decreases until it is less than V_{th}, pinching off the channel. This occurs when

$$V_{DS} > \frac{V_{GS} - V_{th}}{1+\alpha} = V_{DS\,\mathrm{sat}}. \tag{7}$$

When V_{DS} exceeds the saturation voltage, a channel no longer exist all the way to the drain. This is known as the saturation or "pinch-off" region. Additional voltage is dropped across a narrow depleted (not inverted) region near the drain and the drain current remains near its maximum value:

$$I_{DS\,\mathrm{sat}} = \pm\frac{W}{L}\mu' C_{ox}'\left[\frac{(V_{GS} - V_{th})^2}{2(1+\alpha)}\right] \tag{8}$$