EE 531
Winter 2007

Exam 1 Solutions

1. For $N_a = 10^{16}$, $\tau_n^1 \cong \tau_p^1 \cong 2 \times 10^{-4}\,s$ (extrapolating plot on pg. 6)

$L_n^1 \cong 800\,\mu m$ $\quad D_n^1 \cong 32\,cm^2/s$, $D_p^1 \cong 10\,cm^2/s$ (pg. 5 of 2nd set of 482 notes)

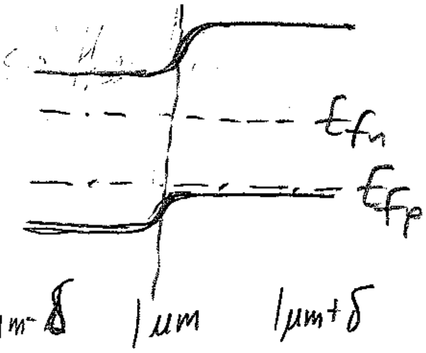
$N_a = 10^{19}\,cm^{-3}$ $\quad \tau_n^2 \cong \tau_p^2 \cong 8 \times 10^{-8}\,s$

$L_n^2 \cong 5\,\mu m$ $\quad D_n^2 = 3\,cm^2/s$, $D_p^2 \cong 1.5\,cm^2/s$

(a) In lightly-doped region, $x_1 = 1\,\mu m \ll L_n$ so negligible recombination (nearly linearly variation of minority carrier concentration, n, w/depth)

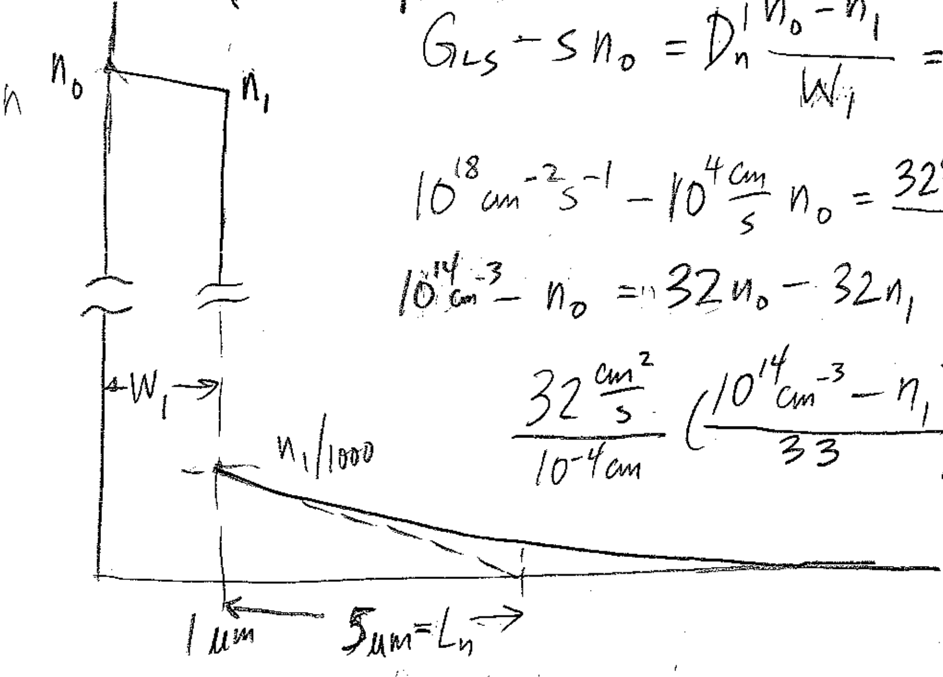
At interface between regions, quasi-Fermi levels are nearly constant (finite recombination), so there is step in n, p

p increases by factor of $1000 = \frac{10^{19}}{10^{16}}$, while n decreases by factor of 1000



close-up of region near interface

Assume LLI (Diffusion approx):



$$G_{LS} - s n_0 = D_n^1 \frac{n_0 - n_1}{W_1} = D_n^2 \frac{n_1}{1000 \cdot L_n^2}$$

$$10^{18}\,cm^{-2}s^{-1} - 10^4 \frac{cm}{s} n_0 = \frac{32 \frac{cm^2}{s} (n_0 - n_1)}{10^{-4}\,cm}$$

$$10^{14}\,cm^{-3} - n_0 = 32 n_0 - 32 n_1 \Rightarrow n_0 = \frac{10^{14}\,cm^{-3} + 32 n_1}{33}$$

$$\frac{32 \frac{cm^2}{s}}{10^{-4}\,cm} \left(\frac{10^{14}\,cm^{-3} - n_1}{33} \right) = \frac{3\,cm^2/s}{0.5\,cm} n_1$$

$$9700 (10^{14}\,cm^{-3} - n_1) = 6 n_1$$

$$\Rightarrow \underline{n_1 \cong 10^{14}\,cm^{-3}}$$

$$\underline{n_0 \cong 10^{14}\,cm^{-3}}$$

Low level injection

(b) $J_p^{diff}(0) = -\frac{D_p'}{D_n'} J_n^{diff}(0)$

$$J_p^{drift}(0) = J_p^{total}(0) - J_p^{diff}(0) = -J_n^{total}(0) - J_p^{diff}(0)$$

$$J_n^{diff}(0) \cong J_n^{diff}(1\mu m) = -q\frac{D_n^2}{L_n^2}\left(\frac{n_1}{1000}\right) = -\frac{(1.6\times10^{-19}c)(3\frac{cm^2}{s})}{5\times10^{-4}\,cm} 10^{10}\,cm^{-3}$$

$$= -9.6\times10^{-6}\frac{A}{cm^2}$$

$$J_p^{diff} = +\frac{10}{32}\,9.6\times10^{-6}\frac{A}{cm^2} = \underline{\underline{3.0\times10^{-6}\frac{A}{cm^2}}}$$

$$J_p^{drift} = 9.6\times10^{-6}A/cm^2 - 3.0\times10^{-6}A/cm^2 = \underline{\underline{6.6\times10^{-6}\frac{A}{cm^2}}}$$

2 (a) $N_c = \underset{\uparrow\,\#\text{ of minima}}{8}\left(2\left(\frac{m^* kT}{2\pi\hbar^2}\right)^{3/2}\right)$ $\qquad A = \frac{\hbar^2}{2m^*} \Rightarrow \frac{m^*}{\hbar^2} = \frac{1}{2A}$

$$= 16\left(\frac{kT}{4\pi A}\right)^{3/2} = 2\left(\frac{kT}{\pi A}\right)^{3/2} \qquad \frac{m^*}{h^2} = \frac{1}{8\pi^2 A}$$

(b) Isotropic elastic scattering, so expected value of $k = 0$ after scattering: $\frac{1}{\tau_m} = \frac{1}{\tau} = \int_{k'} S(k,k')dk'$

$$= S_0\, g_c(E) = S_0\left[8\,\frac{1}{2}\,\frac{4\pi}{h^3}(2m^*)^{3/2}(E-E_c)^{1/2}\right]$$

# of minima; spin

$$= S_0\, 16\pi\left(\frac{2}{8\pi^2 A}\right)^{3/2}(E-E_c)^{1/2}$$

$$= S_0\,\frac{2}{\pi^2}A^{-3/2}(E-E_c)^{1/2}$$

$$\tau_m(E) = \frac{\pi^2}{2S_0}A^{3/2}(E-E_c)^{-1/2}$$

3. $\tau_m = K_m T_e^{-1/2}$, $\tau_E = K_E T_e^{1/2}$

Assume spatially uniform and steady state ($\nabla_r \Rightarrow 0$, $\frac{\partial}{\partial t} \Rightarrow 0$)

$$\frac{d\langle p\rangle}{dt} = 0 = -q\mathcal{E} - \frac{\langle p\rangle}{\tau_m} \qquad \langle p\rangle = -q\mathcal{E}\tau_m = -q\mathcal{E}K_m T_e^{-1/2}$$

$$\frac{d\langle E\rangle}{dt} = 0 = -\frac{1}{m^*} q\mathcal{E}\cdot\langle p\rangle - \frac{\langle E\rangle - E_0}{\tau_E}$$

$$\langle E\rangle = \frac{3}{2}kT_e + \frac{\langle p\rangle^2}{2m^*} = E_0 - \tau_E\left(\frac{1}{m^*}q\mathcal{E}\cdot\langle p\rangle\right)$$

$$\frac{3}{2}kT_e + \frac{q^2\mathcal{E}^2K_m^2}{2m^*T_e} - E_0 - \frac{K_E\cancel{T_e^{1/2}}}{m^*}q^2\mathcal{E}^2K_m\cancel{T_e^{-1/2}} = 0$$

$$\frac{3}{2}kT_e^2 - \left(E_0 + \frac{q^2\mathcal{E}^2K_EK_m}{m^*}\right)T_e + \frac{q^2\mathcal{E}^2K_m^2}{2m^*} = 0$$

$\frac{3}{2}kT_e > E_0$

$$T_e = \frac{(E_0 + \alpha\mathcal{E}^2) + \sqrt{(E_0+\alpha\mathcal{E}^2)^2 - 6k(\beta\mathcal{E}^2)}}{3k} \qquad \alpha = \frac{q^2K_EK_m}{m^*} \qquad \beta = \frac{q^2K_m^2}{2m^*}$$

$$\langle v\rangle = \frac{\langle p\rangle}{2m^*} = -\frac{q\mathcal{E}K_m}{m^*}T_e^{-1/2}$$

Look at limiting cases: $\mathcal{E}$ small ($\langle E\rangle \cong E_0$)

$$T_e \cong \frac{E_0 + \alpha\mathcal{E}^2 + E_0 + \alpha\mathcal{E}^2 - \frac{6k(\beta\mathcal{E}^2)}{2(E_0 \cancel{+\alpha\mathcal{E}^2})}}{3k} \cong \underbrace{\frac{2E_0}{3k}}_{T_l} + \left(\frac{2\alpha}{3k} - \frac{\beta}{E_0}\right)\mathcal{E}^2$$

$\cong T_l$, $\underline{v \propto \mathcal{E}}$

For $\mathcal{E}$ large: ($\langle E\rangle \gg E_0$)

$$T_e \cong \frac{2\alpha\mathcal{E}^2}{3k} \Rightarrow \langle v\rangle = -\frac{q\cancel{\mathcal{E}}K_m}{m^*}\left(\frac{3k}{2\alpha}\right)^{1/2}\frac{1}{\cancel{\mathcal{E}}} = \text{constant}$$

(velocity saturation)

4. $\phi_s = \chi_s + \frac{E_g}{q} - \frac{kT}{q}\ln\frac{N_v}{N_a} = \chi_s + 1.12\,V - 0.026\,V \ln\left(\frac{1.8\times10^{19}}{5\times10^{17}}\right)$

$= \chi_s + 1.03\,V$

$|\psi_B| = \frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) = 0.026\,V \ln\left(\frac{5\times10^{17}}{10^{10}}\right) = 0.46\,V$

(a) $-Q_S = \sqrt{2K_s\epsilon_0 kTN_a}\left[\frac{q\psi_s}{kT} + \frac{n_i^2}{N_a^2}e^{q(\psi_s - V_{CB})/kT}\right]^{1/2}$

$= \left(2 * 11.8 * 8.854\times10^{-14}\frac{F}{cm} * 0.026\,eV * 5\times10^{17}\,cm^{-3}\right)^{1/2}\left[\frac{1V + 2(0.46V) + 0.1V}{0.026V} + e^{0.1V/0.026V}\right]^{1/2}$

$= \left(2.7\times10^4\ \frac{F\cdot eV}{cm^4}\right)^{1/2}\left(77.7 + 46.8\right)^{1/2}$

$1\,eV = 1.6\times10^{-19}\,CV \Rightarrow \frac{F\cdot eV}{cm^4} = \frac{C}{\cancel{V}}\frac{C\cancel{V}}{cm^4}(1.6\times10^{-19})$

$= 1.6\times10^{-19}\ \frac{C^2}{cm^4}$

$Q_s = -\left(1.6\times10^{-19} \cdot 2.7\times10^4\right)^{1/2}(124.5)^{1/2}\ \frac{C}{cm^2}$

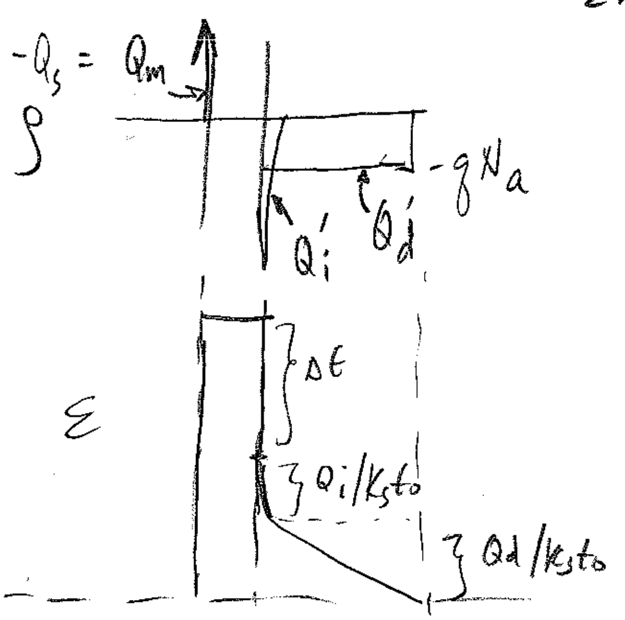
$= -7.3\times10^{-7}\ C/cm^2$

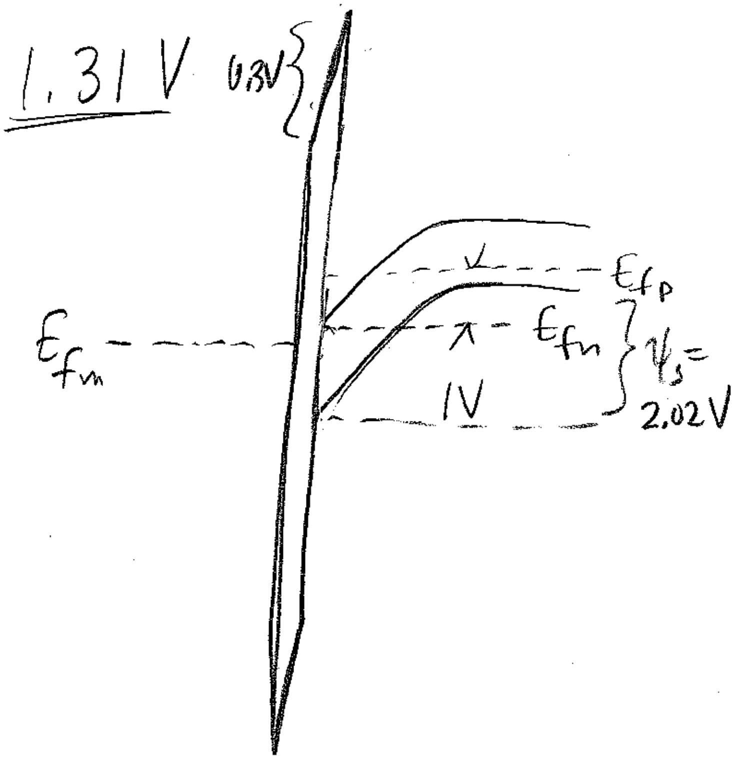
$C_{ox}' = \frac{K_{ox}\epsilon_0}{x_{ox}} = \frac{3.9\times 8.854\times10^{-14}F/cm}{1.5\times10^{-7}\,cm} = 2.3\times10^{-6}\frac{F}{cm^2}$

$V_{GB} = \Phi_{MS} + \psi_s - \frac{Q_s'}{C_{ox}'}$

$= \left[\chi_s - (\chi_s + 1.03V)\right] + \left(1V + 2(0.46V) + 0.1V\right) + \frac{7.3\times10^{-7}C/cm^2}{2.3\times10^{-6}F/cm^2}$

$= -1.03 + 2.02 + \frac{0.73}{2.3} = \underline{1.31\ V}$





(b) $C'_{GB} = \dfrac{C'_{ox} C'_s}{C'_{ox} + C'_s}$ $\qquad C'_{ox} = 2.3\times10^{-6}\ F/cm^2$

$$C_s' = \left|\frac{dQ_s}{d\psi_s}\right|$$

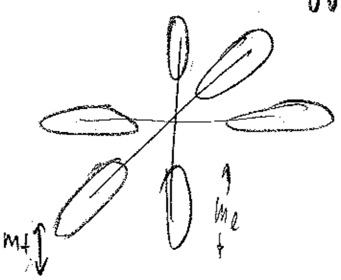
$$\left|\frac{dQ_s}{d\psi_s}\right| = \sqrt{2K_s\epsilon_0 kTN_a}\ \frac{1}{2}\left(\frac{q\psi_s}{kT} + e^{q(\psi_s - V_{CB} - 2|\psi_B|)/kT}\right)^{-1/2}\left[\frac{q}{kT} + \frac{q}{kT}e^{\frac{(\psi_s - V_{CB} - 2|\psi_B|)}{kT}}\right]$$

$$= \frac{-Q_s'}{2(124.5)(0.026V)}\left[1 + \underbrace{e^{0.1/0.026}}_{46.8}\right] = 5.4\times10^{-6}\ \frac{F}{cm^2}$$

$$C'_{GB} = \frac{(2.3)(5.4)}{2.3+5.4}\times10^{-6}\ F/cm^2 = 1.61\times10^{-6}\ F/cm^2$$

(c) $\mathcal{E}_s = -\dfrac{Q_s}{K_s\epsilon_0} = \dfrac{+7.3\times10^{-7}\ C/cm^2}{11.8 * 8.854\times10^{-14}\ F/cm} = 7.0\times10^5\ \dfrac{V}{cm}$

$m_\perp$ is different for different minima $\quad m_\perp = \begin{cases} m_t = 0.19m_0 & \text{for 4 minima (4-fold)} \\ m_l = 0.92m_0 & \text{for 2 minima (2-fold)} \end{cases}$



$$\left[\frac{3hq\mathcal{E}_s}{4\sqrt{2m_0}}\right]^{2/3} = \left[\frac{3^2(6.63\times10^{-34}J\cdot s)^2(1.6\times10^{-19}C)^2\left(7\times10^5\frac{V}{cm}\right)^2}{4^2\ (2 * 9.11\times10^{-31}kg)}\right]^{1/3}$$

$$= 1.2\times10^{-21}J\left[\left(\frac{\cancel{kg}\,m^2}{\cancel{s^2}}\right)\left(\frac{\cancel{s^2}\,cm^{-2}}{\cancel{kg}}\right)\right]^{1/3}$$

$$= 2.6\times10^{-20}\ J = 0.16\,eV$$

$$E_j = 0.16eV\left(\frac{m_0}{m_\perp}\right)^{1/3}\left(j + \frac{3}{4}\right)^{2/3}$$

$$\left.\begin{aligned} E_0^2 &= 0.16\left(\frac{1}{0.92}\right)^{1/3}(0.75)^{2/3} = 0.14\,eV \\ E_0^4 &= 0.16\left(\frac{1}{0.19}\right)^{1/3}(0.75)^{2/3} = 0.23\,eV \\ E_1^2 &= 0.16\left(\frac{1}{0.92}\right)^{1/3}(1.75)^{2/3} = 0.24\,eV \end{aligned}\right\}$$

All of these values are much greater than kT, so inversion charge is substantially reduced.

Can go back to re-evaluate $\mathcal{E}_s \cong -\dfrac{Q_d}{K_s\epsilon_0} = \sqrt{\dfrac{2qN_a\psi_s}{K_s\epsilon_0}} = 5.6\times10^5\ \dfrac{V}{cm}$

Thus, $E_0^2 = 0.14\,eV\left(\frac{5.6}{7.0}\right)^{2/3} = 0.12eV$, $E_0^4 = 0.20eV$, $E_1^2 = 0.21eV$

It is reasonable to consider just the lowest sub-band as $E_0^4 - E_0^2 = 1.09eV \cong 3.5kT$. For 2-fold band $m^* = m_t$ in plane of 2DEG

2D Density of states is $\frac{4\pi\sqrt{m_t^2}}{h^2}$ per sub-band

$$Q_i^{QM} \cong 2q\int_{E_0^2}^{\infty} g(E)\, e^{-(E-E_f)/kT}\, dE \qquad E_0^2 \quad q(\psi_s - V_{CB} - 2|\psi_B|)$$

$$\cong 2q\int_0^{\infty} \frac{4\pi(m_t)}{h^2}\, e^{-(0.14-0.1)/kT}\, e^{-E/kT}\, dE$$

$$= \frac{8q\pi m_t}{h^2}(kT)\, e^{-0.04/0.026}$$

$$= \frac{8(1.6\times10^{-19}C)(\pi)(0.19\times9.11\times10^{-31}\cancel{kg})}{(6.63\times10^{-34}\frac{\cancel{kg}\, m^2}{\cancel{s}})(4.14\times10^{-15}\cancel{eV}\cdot\cancel{s})}(0.026\cancel{eV})(0.21)$$

$$= 1.4\times10^{-3}\, C/m^2 = 1.4\times10^{-7}\, C/cm^2$$

(d) Dominated by 2-fold sub-band $m_{\parallel} \cong m_t \cong 0.19 m_0$

Can consider other sub-bands DOS $\propto g\sqrt{m_x m_y}$ 2 of 4 aligned with in-plane field

$$m^* = \left[\frac{\frac{1}{m_t}\left(2 + 2\sqrt{\frac{m_l}{m_t}}\, e^{-(E_0^4-E_0^2)/kT} + 2e^{-(E_1^2-E_0^2)/kT}\right) + \frac{1}{m_l}\left(2\sqrt{\frac{m_l}{m_t}}\, e^{-\frac{(E_0^4-E_0^2)}{kT}}\right)}{2 + \underbrace{4\sqrt{\frac{m_l}{m_t}}\, e^{-(E_0^4-E_0^2)/kT}}_{0.07} + \underbrace{2e^{-(E_1^2-E_0^2)/kT}}_{0.02}}\right]^{-1}$$

$$= 0.20\, m_0$$