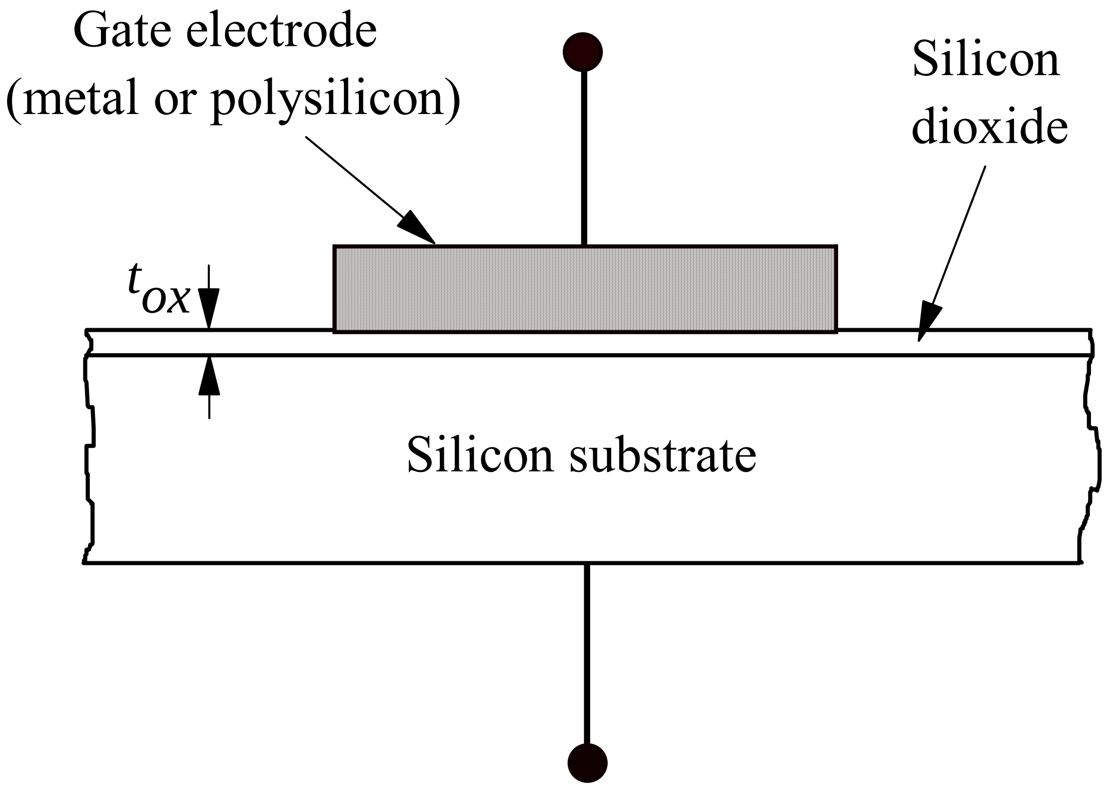
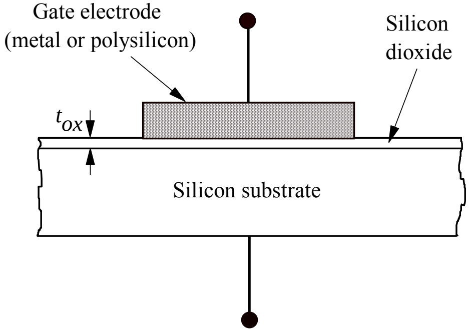
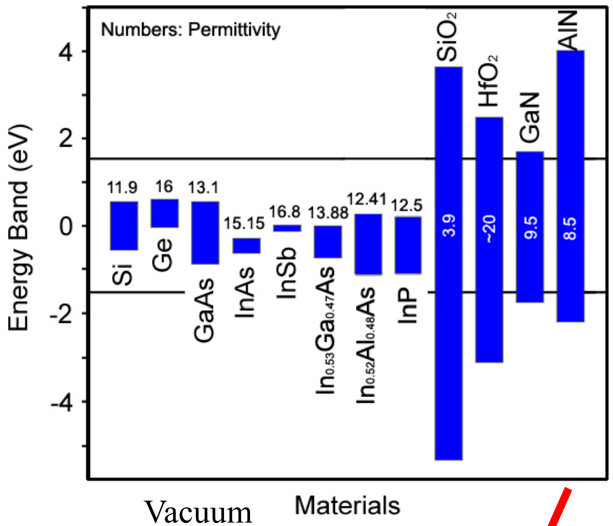
Let’s Start with the MOS Capacitor



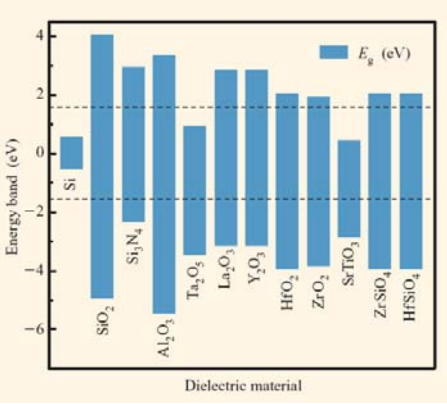
Metal-Oxide-Semiconductor (MOS)

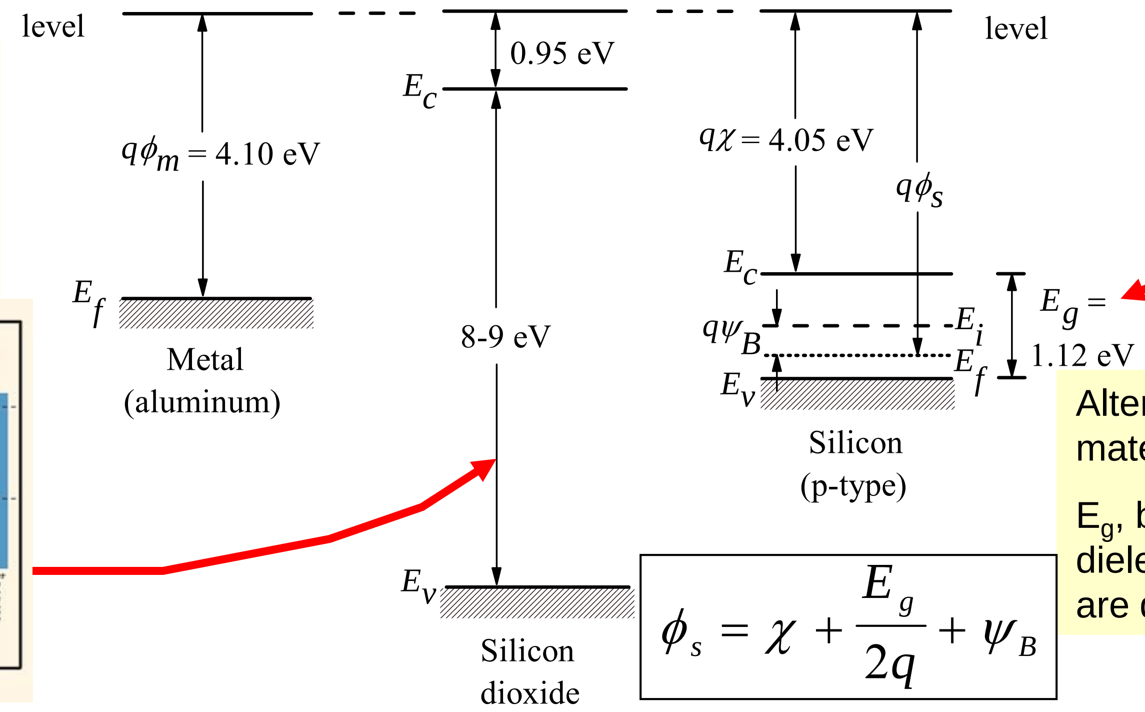




High-k:

E_g and barrier height are different from SiO_2





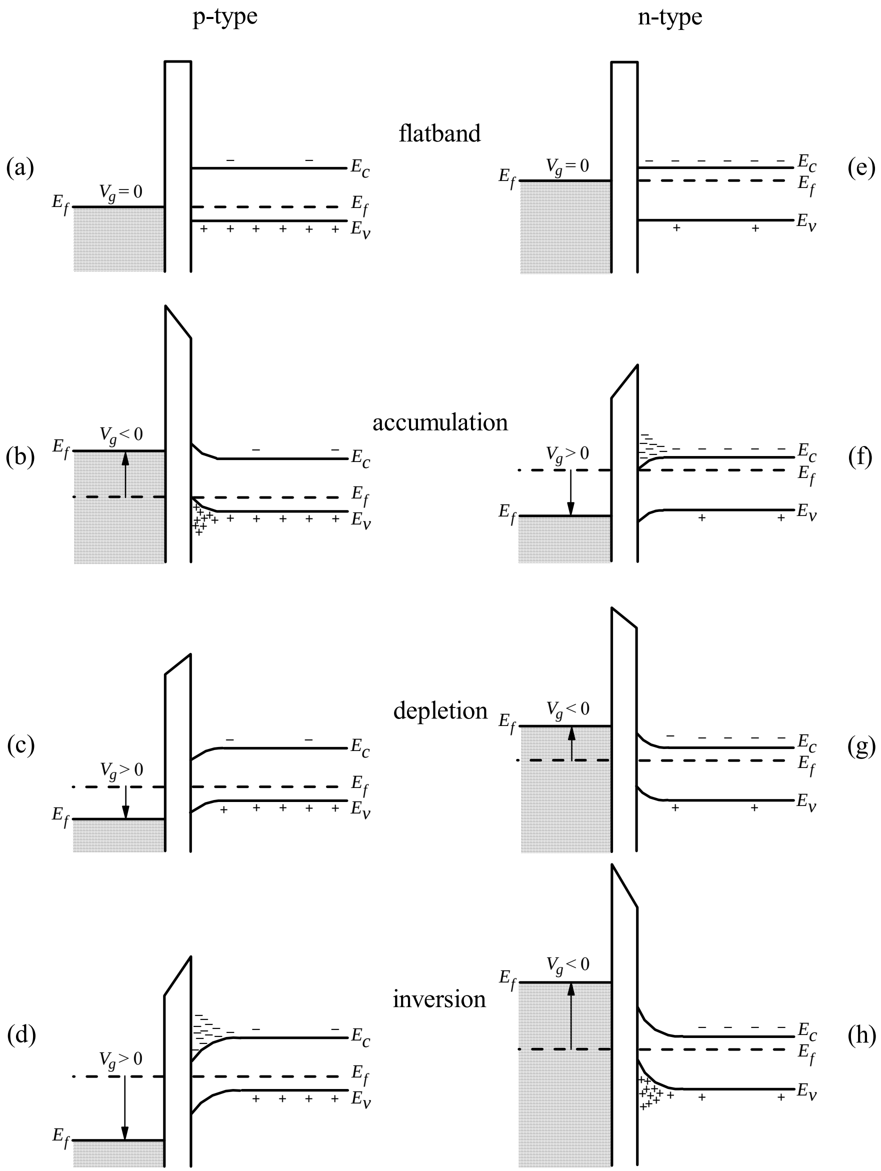
$$\phi_s = \chi + \frac{E_g}{2q} + \psi_B$$

Alternative channel materials:

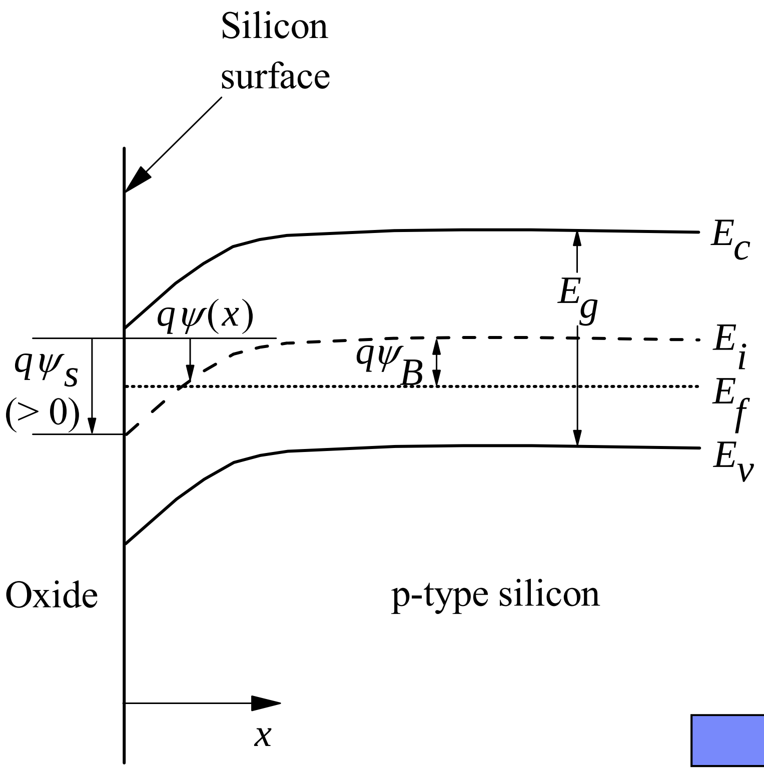
E_g, barrier height, and dielectric constant are different from Si

Accumulation, Depletion, Inversion

Assume $\phi_m=\phi_s$:



Poisson's Equation



$$\frac{d^2\psi}{dx^2} = -\frac{d\mathrm{E}}{dx} = -\frac{q}{\varepsilon_{si}}\left[p(x) - n(x) + N_d^{\,+}(x) - N_a^{\,-}(x)\right]$$

$$p(x) = n_i e^{(E_i - E_f)/kT} = n_i e^{q(\psi_B - \psi)/kT} = p_0 e^{-q\psi/kT}$$

$$n(x) = n_i e^{(E_f - E_i)/kT} = n_i e^{q(\psi - \psi_B)/kT} = n_0 e^{q\psi/kT}$$

charge neutrality:

(assuming uniform doping)

$$\left[p(\psi = 0) - n(\psi = 0) + N_d^{\,+} - N_a^{\,-}\right] = 0$$

$$N_d^{\,+} - N_a^{\,-} = -p(\psi = 0) + n(\psi = 0)$$

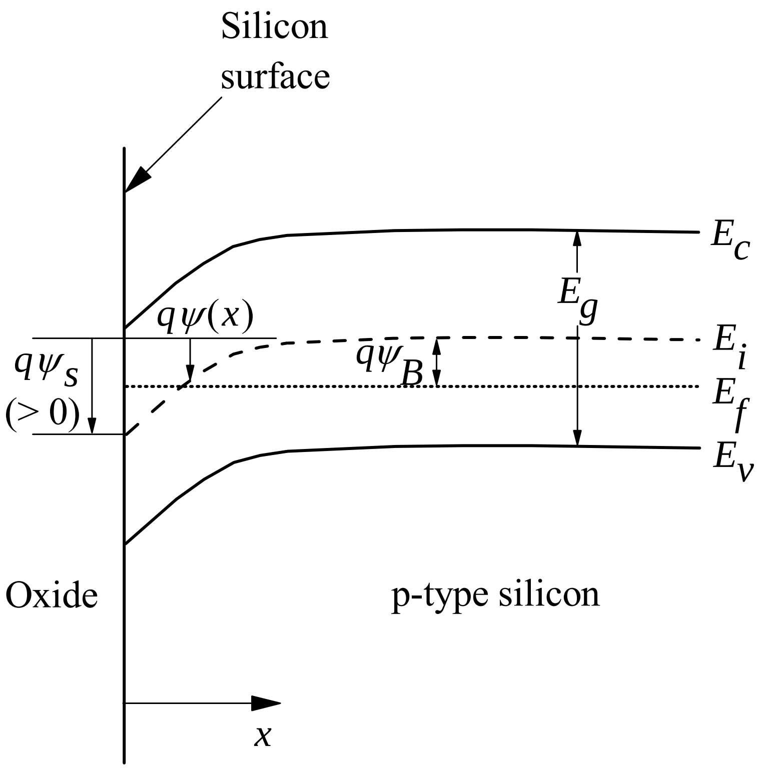
$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_{si}}\left[\underbrace{p_0(e^{-q\psi/kT}}_{p(x)}\ \underbrace{-1)}_{N_d^{\,+} - N_a^{\,-}} - \underbrace{n_0(e^{q\psi/kT}}_{n(x)}\ \underbrace{-1)}_{N_d^{\,+} - N_a^{\,-}}\right]$$

For N_a doped substrate
($p_0 = N_a$,
$n_0 = n_i^2/N_a$)

$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_{si}}\left[N_a\left(e^{-q\psi/kT} - 1\right) - \frac{n_i^2}{N_a}\left(e^{q\psi/kT} - 1\right)\right]$$

(assuming complete ionization)

Poisson's Equation



$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_{si}}\left[N_a\left(e^{-q\psi/kT}-1\right)-\frac{n_i^2}{N_a}\left(e^{q\psi/kT}-1\right)\right]$$

Use a trick :

$$\frac{1}{2}\frac{\partial}{\partial x}\left[\left(\frac{\partial\psi}{\partial x}\right)^2\right] = \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x^2}$$

$$E = -\frac{\partial\psi}{\partial x}$$

$$\frac{1}{2}\int_{x=\infty}^{x} d\left(\frac{d\psi}{dx}\right)^2 = \int_{x=\infty}^{x} d\phi\left(-\frac{qN_a}{\varepsilon_{si}}\right)\left[\left(e^{-q\psi/kT}-1\right)-\frac{n_i^2}{N_a^2}\left(e^{q\psi/kT}-1\right)\right]$$

Poisson’s Equation

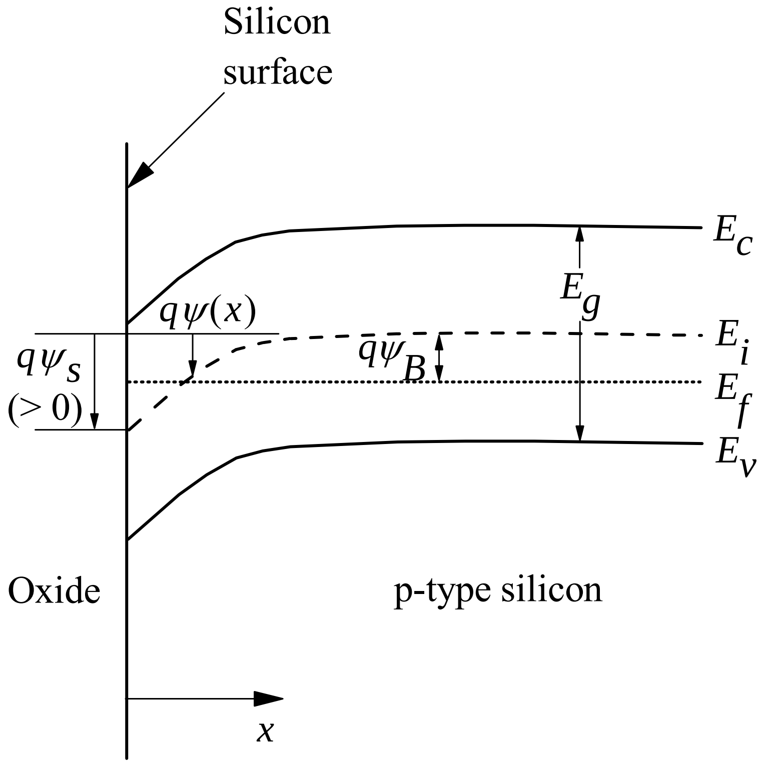
$$\frac{1}{2}\int_{x=\infty}^{x} d\left(\frac{d\psi}{dx}\right)^{2} = \int_{x=\infty}^{x} d\phi\left(-\frac{qN_a}{\varepsilon_{si}}\right)\left[\left(e^{-q\psi/kT}-1\right)-\frac{n_i^{2}}{N_a^{2}}\left(e^{q\psi/kT}-1\right)\right]$$

$$E = -\frac{\partial\psi}{\partial x}$$

Boundary conditions:

$$\phi\left(x\to\infty\right)=0$$

$$\left.\frac{d\psi}{dx}\right|_{x\to\infty} = 0$$



Solving Poisson’s Equation

$$\mathrm{E}^2(x) = \left(\frac{d\psi}{dx}\right)^2 = \frac{2kTN_a}{\varepsilon_{si}}\left[\left(\underbrace{e^{-q\psi/kT}}_{p(x)} + \underbrace{\frac{q\psi}{kT}}_{} \underbrace{-1}_{p(x)}\right) + \frac{n_i^2}{N_a^2}\left(\underbrace{e^{q\psi/kT}}_{n(x)} \underbrace{-\frac{q\psi}{kT}}_{} \underbrace{-1}_{n(x)}\right)\right] \quad \boxed{N_d^+ - N_a^-}$$

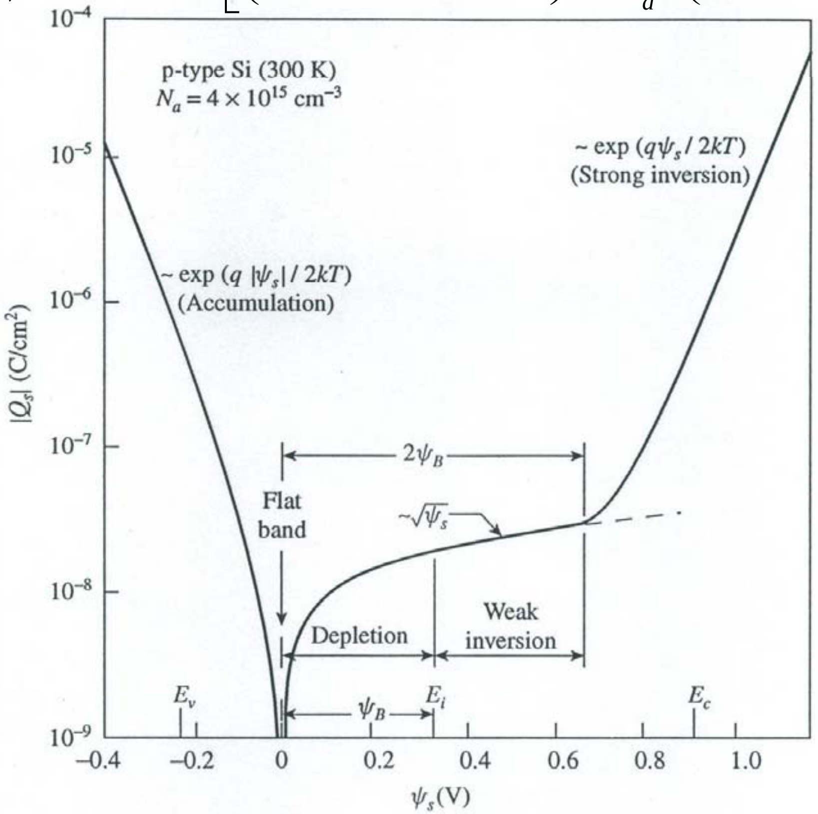
$$Q_s = -\varepsilon_{si}\mathrm{E}_s = \pm\sqrt{2\varepsilon_{si}kTN_a}\left[\left(e^{-q\psi_s/kT} + \frac{q\psi_s}{kT} - 1\right) + \frac{n_i^2}{N_a^2}\left(e^{q\psi_s/kT} - \frac{q\psi_s}{kT} - 1\right)\right]^{1/2}$$

Term related to the Debye length:

$$L_D \equiv \sqrt{\frac{\varepsilon_{si}kT}{q^2 N_d}}$$

L_D=41 nm for N_d = 10^{16}cm^{-3} at 300K

For details, see C.Y. Chang, S.M. Sze, ULSI Devices, Wiley, Chapter 3.

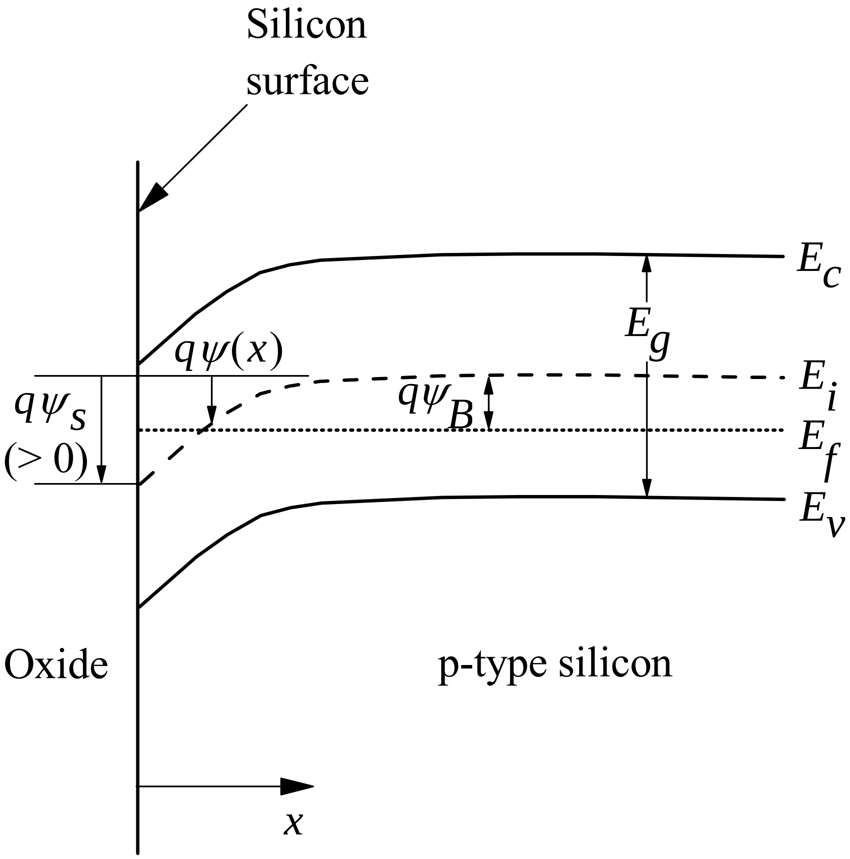


What is the value of n_i for Si?

Depletion Charge, Inversion Charge

$$Q_s = \underbrace{Q_d}_{\text{Depletion Charge}} + \underbrace{Q_i}_{\text{Inversion Charge}}$$

Condition for Strong Inversion



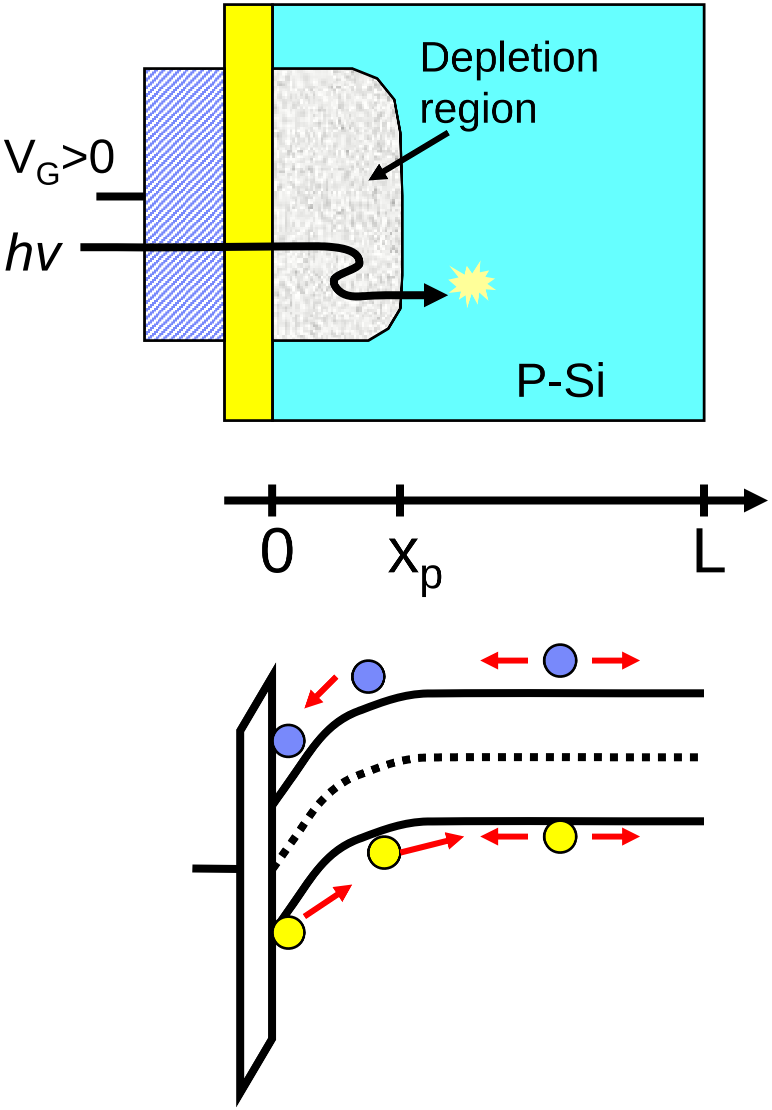
$$\psi_s(inv) = 2\psi_B = 2\frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right)$$

i.e., $(n_i^2/N_a^2)\exp(q\psi_s/kT) = 1$.

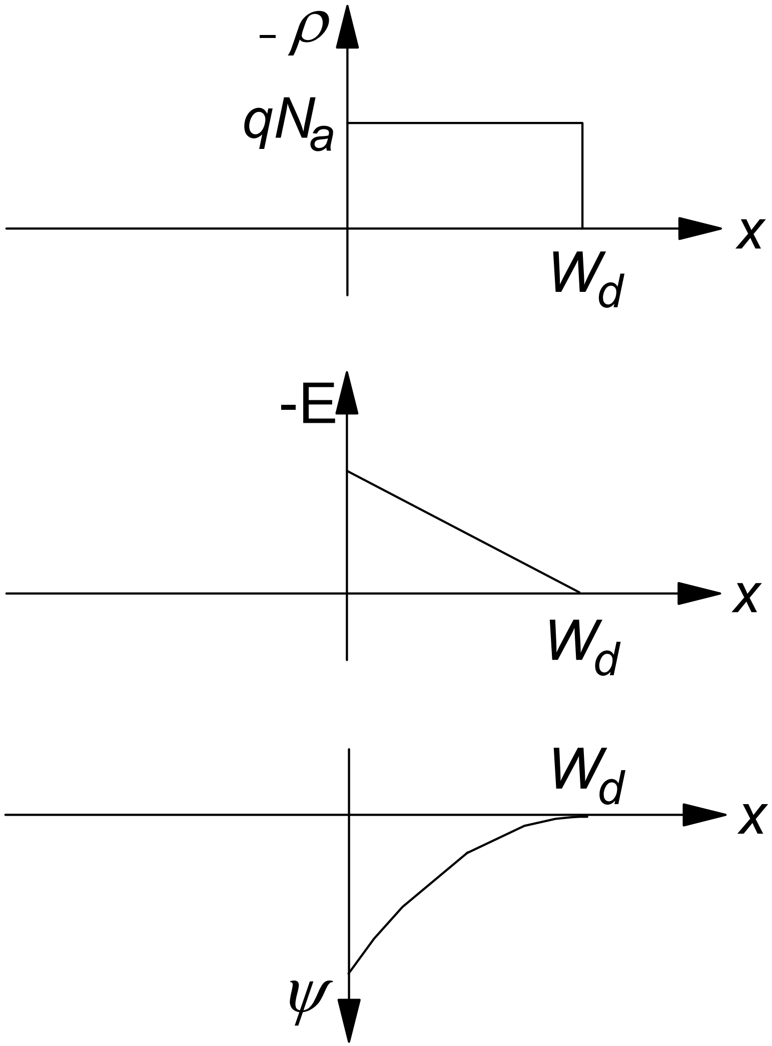
And the electron concentration at the surface equals the hole concentration in the bulk Si.

Carrier Generation Transient – Example: Photo-Generation

- **Electrons generated in the depletion region will be collected in the potential well**
- **Electrons generated in the neutral region will**
  - Recombine with holes
  - Diffuse to depletion region and get collected in the potential well if it is within the diffusion length of the minority carriers
- **Holes will be collected in the substrate**
- **How many of the photo-generated carriers are collected depends on:**
  - Diffusion length of minority carriers
  - Location and length of the depletion region



Depletion Approximation: 1-D Uniform Doping



$$Q_d = -qN_aW_d$$

$$E = -qN_a(W_d - x)/\varepsilon_{si}$$

$$\psi = qN_a(W_d - x)^2/2\varepsilon_{si}$$

$$\Rightarrow \quad \psi_s = qN_aW_d^2/2\varepsilon_{si}$$

$$W_d = \sqrt{\frac{2\varepsilon_{si}\psi_s}{qN_a}}$$

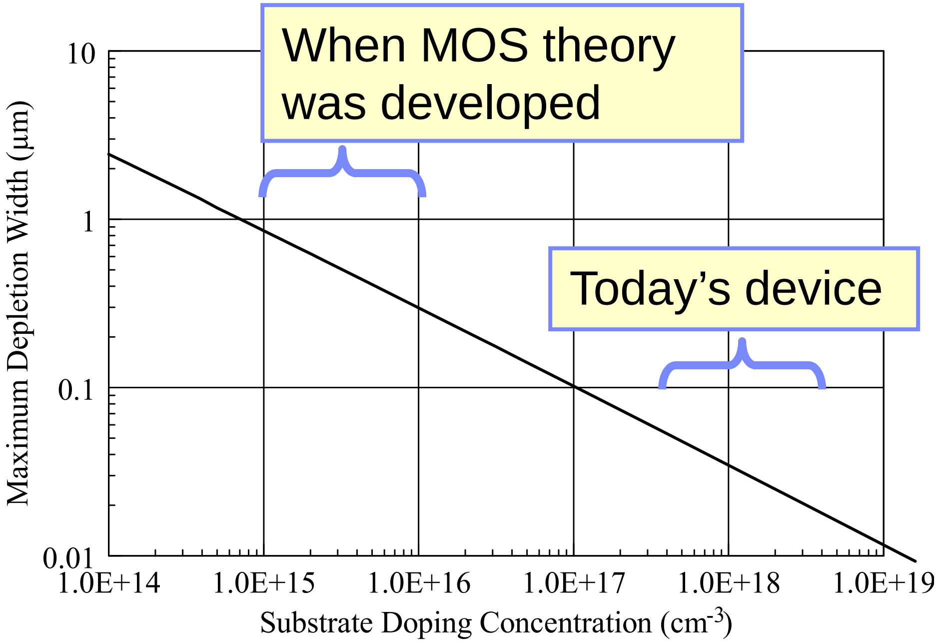
$$\psi = \psi_s\left(1 - \frac{x}{W_d}\right)^2$$

Maximum Depletion Width in MOS (1D Uniform Doping)

In contrast to p-n junctions, W_d reaches a maximum value W_{dm} at the onset of strong inversion when

$$\psi_s = 2\psi_B = 2(kT/q)\ln(N_a/n_i):$$

$$W_{dm} = \sqrt{\frac{4\varepsilon_{si} kT \ln(N_a / n_i)}{q^2 N_a}}$$



When MOS theory was developed

Today’s device

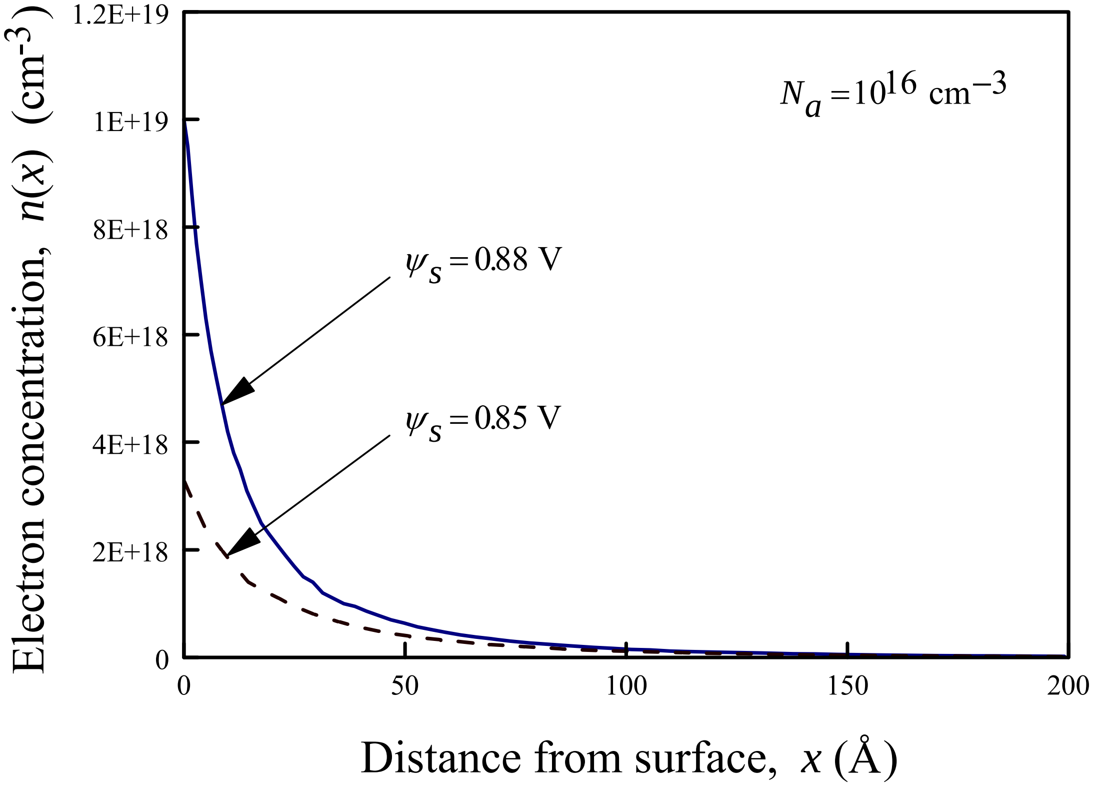
This defines the threshold condition of a MOSFET. W_{dm} also plays a key role in the short-channel scaling of a MOSFET, namely, $L_{min} \propto W_{dm}$.

We will discuss that later

“minimum” channel length

Strong Inversion

$$\frac{d\psi}{dx} = -\sqrt{\frac{2kTN_a}{\varepsilon_{si}}\left(\frac{q\psi}{kT} + \frac{n_i^{\,2}}{N_a^{\,2}} e^{q\psi/kT}\right)}$$



Inversion charge per area:

$$Q_i = -\sqrt{\frac{2\varepsilon_{si} kT n_i^{\,2}}{N_a}} e^{q\psi_s/2kT}$$

Electron conc. at surface:

$$n(0) = \frac{n_i^{\,2}}{N_a} e^{q\psi_s/kT}$$

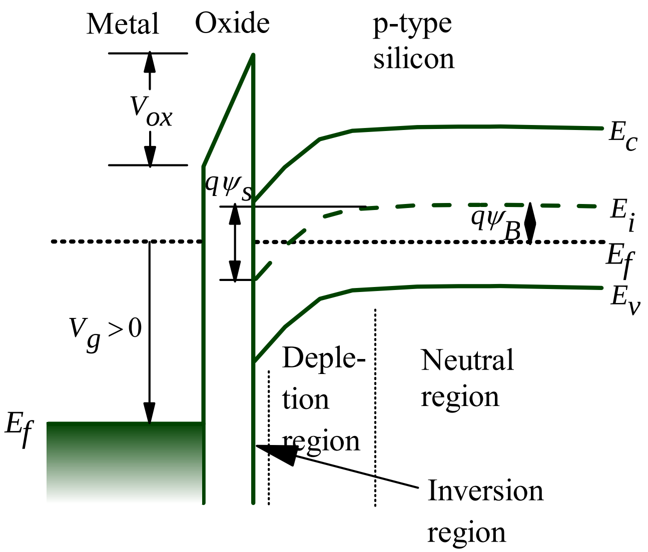
Inversion layer thickness:

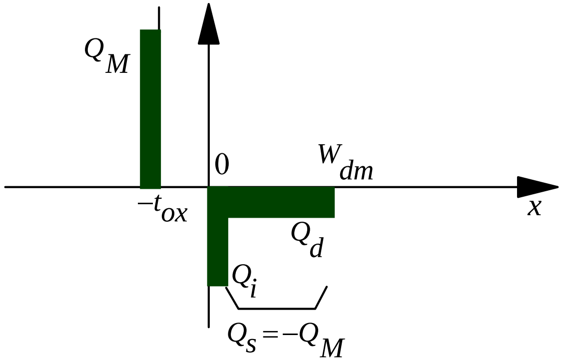
$Q_i/qn(0) = 2\varepsilon_{si}kT/(qQ_i)$

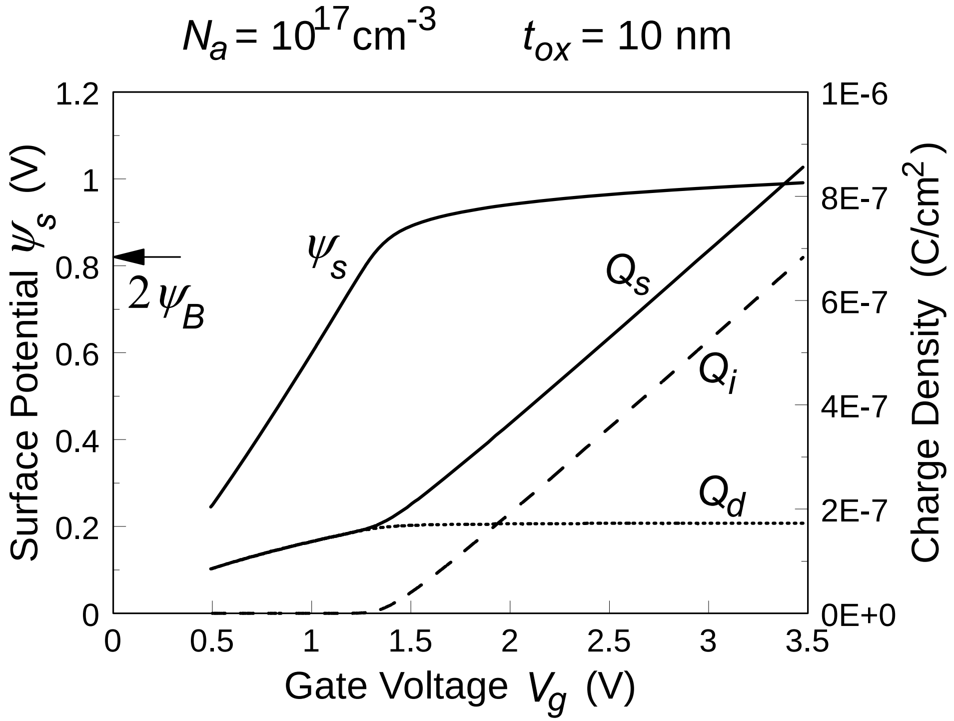
T_{inv}=0.4 nm for $Q_i = 8\times10^{12}$ cm^{-2} at 300K

Large change in carrier concentration with small changes in surface potential (ψ_s) → "pinning" of surface potential at $2\psi_B$

MOSFET Charge and Potential







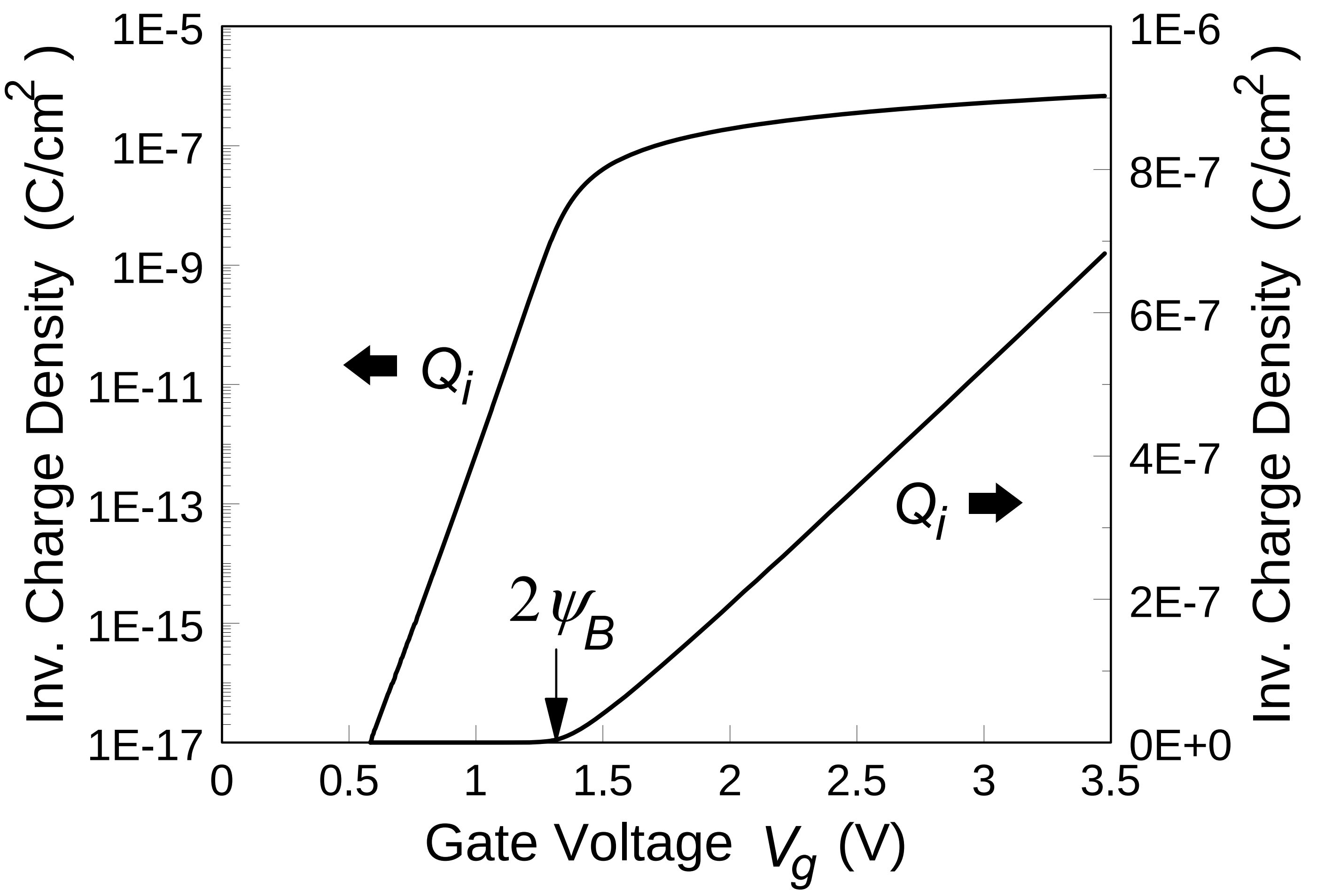
Note: $C_{ox}=\varepsilon_{ox}/t_{ox}$ and $\varepsilon_{ox}E_{ox}=\varepsilon_{si}E_s$

Gate voltage equation (V_{fb}=0):

$$V_g = V_{ox} + \psi_s = \frac{-Q_s}{C_{ox}} + \psi_s$$

$$Q_s = -\varepsilon_{si}E_s = \pm\sqrt{2\varepsilon_{si}kTN_a}\left[\left(e^{-q\psi_s/kT}+\frac{q\psi_s}{kT}-1\right)+\frac{n_i^2}{N_a^2}\left(e^{q\psi_s/kT}-\frac{q\psi_s}{kT}-1\right)\right]^{1/2}$$

Inversion Charge in Log Scale

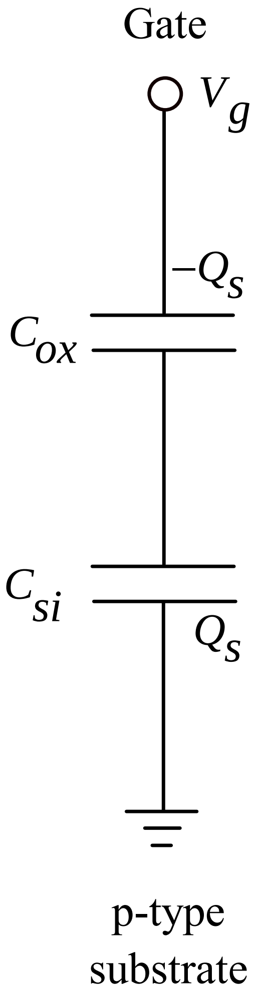


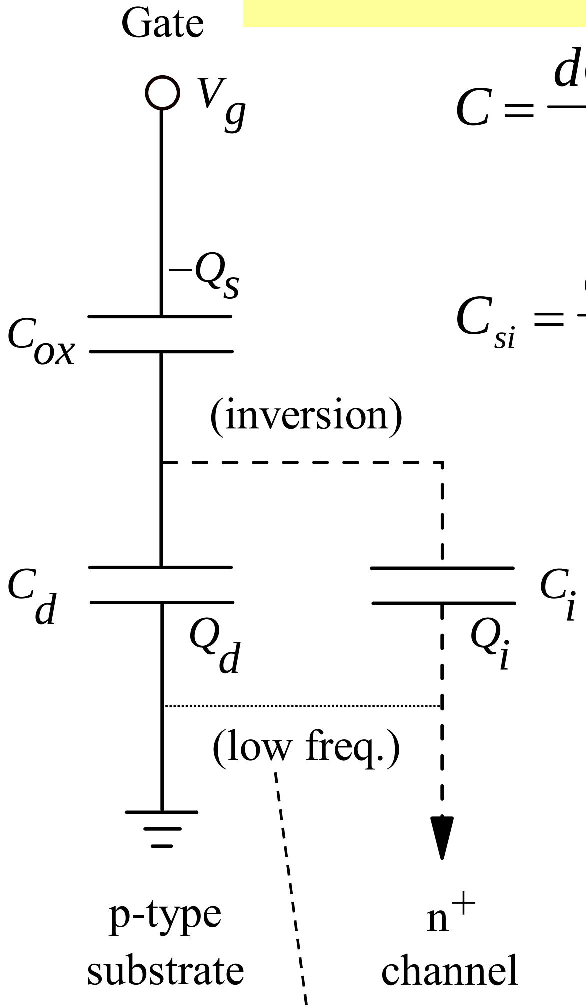
$N_a = 10^{17}\,\text{cm}^{-3}$ $t_{ox} = 10$ nm

MOS Capacitances

Differentiate w.r.t. $-Q_s$

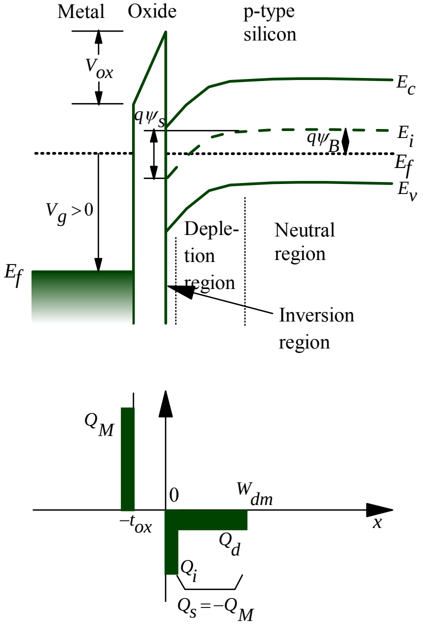
$$V_g = V_{ox} + \psi_s = \frac{-Q_s}{C_{ox}} + \psi_s$$





$$C = \frac{d(-Q_s)}{dV_g}$$

$$C_{si} = \frac{d(-Q_s)}{d\psi_s}$$

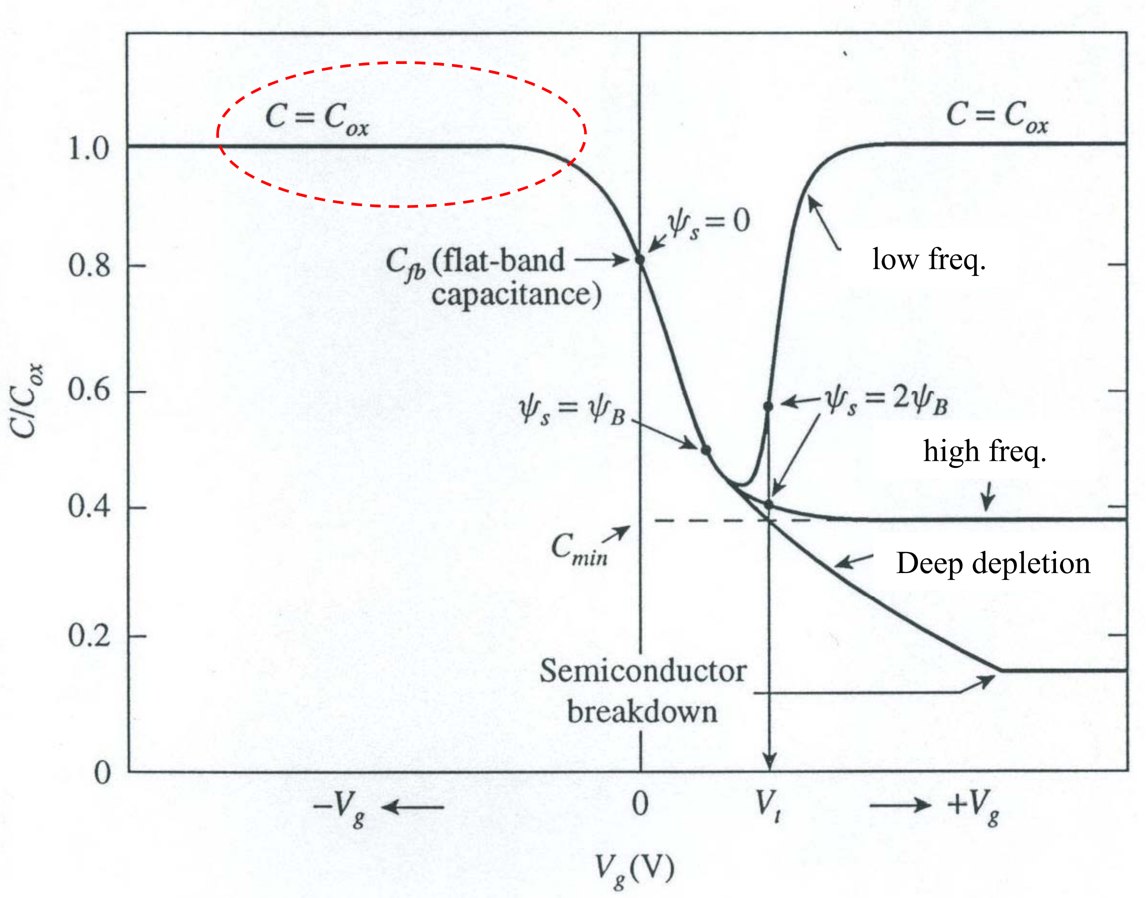


$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{d\psi_s}{d(-Q_s)}$$

$$= \frac{1}{C_{ox}} + \frac{1}{C_{si}}$$

Through carrier generation/recombination

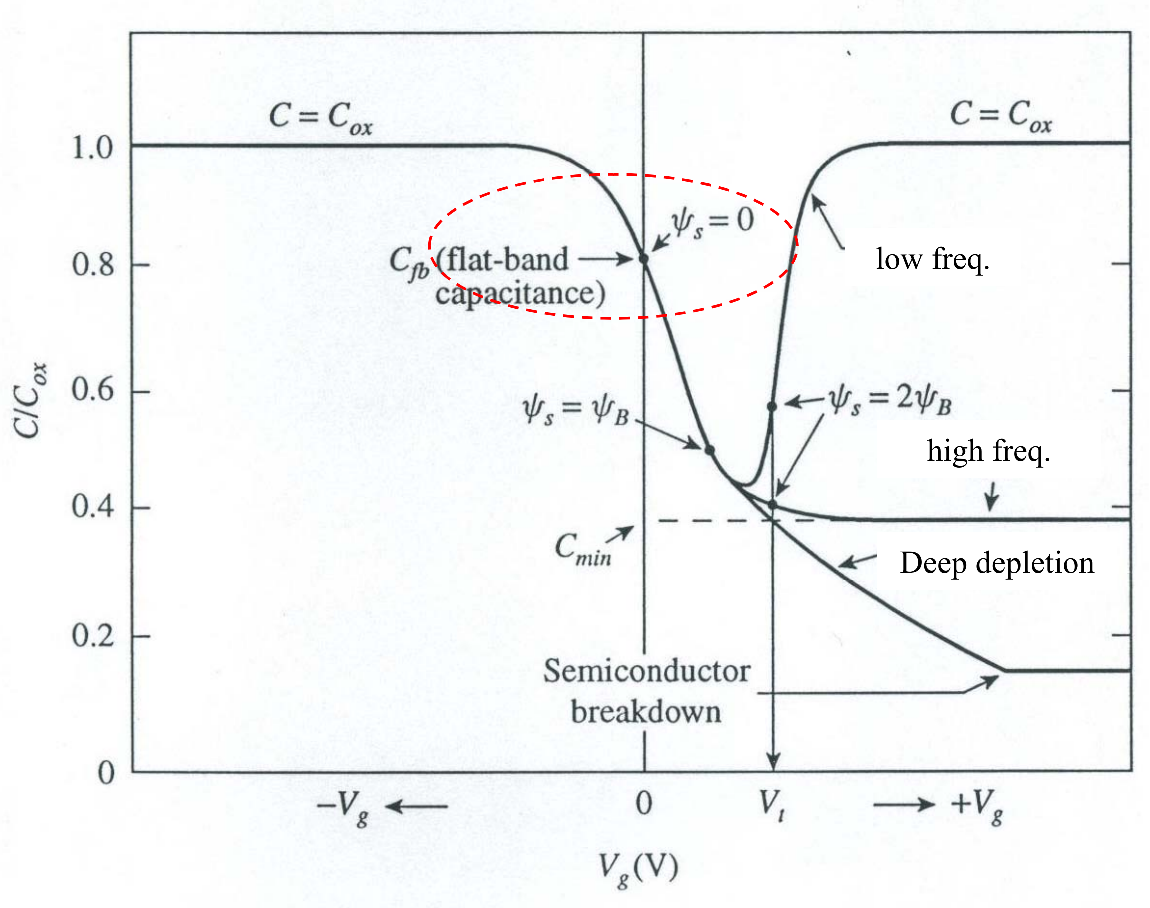
Capacitance-Voltage Characteristics - Accumulation



- In accumulation, $Q_s \propto \exp(-q\psi_s/2kT)$, so $C_{si}=-dQ_s/d\psi_s=(q/2kT)Q_s$ $=(q/2kT)C_{ox}|V_g-\psi_s|$.

$$\frac{1}{C} = \frac{1}{C_{ox}}\left[1+\frac{2kT/q}{|V_g-\psi_s|}\right]$$

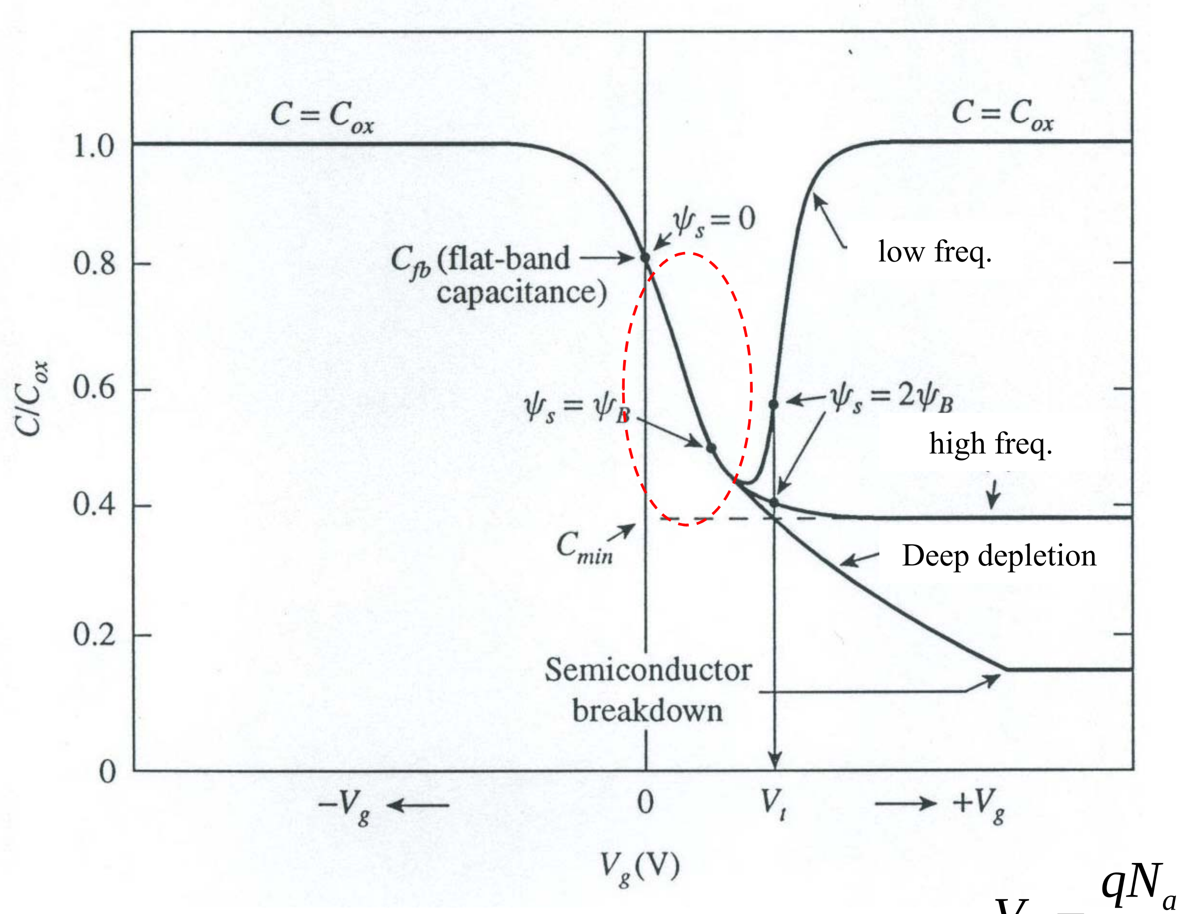
Capacitance-Voltage Characteristics – Flatband



- At flatband voltage, $q\psi_s/kT<<1$, therefore, $Q_s=-(\varepsilon_{si}q^2N_a/kT)^{1/2}\psi_s$.

$$\frac{1}{C_{fb}} = \frac{1}{C_{ox}} + \sqrt{\frac{kT}{\varepsilon_{si}q^2N_a}} = \frac{1}{C_{ox}} + \frac{L_D}{\varepsilon_{si}}$$

Capacitance-Voltage Characteristics – Depletion



- In depletion,

$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_d}$$

where

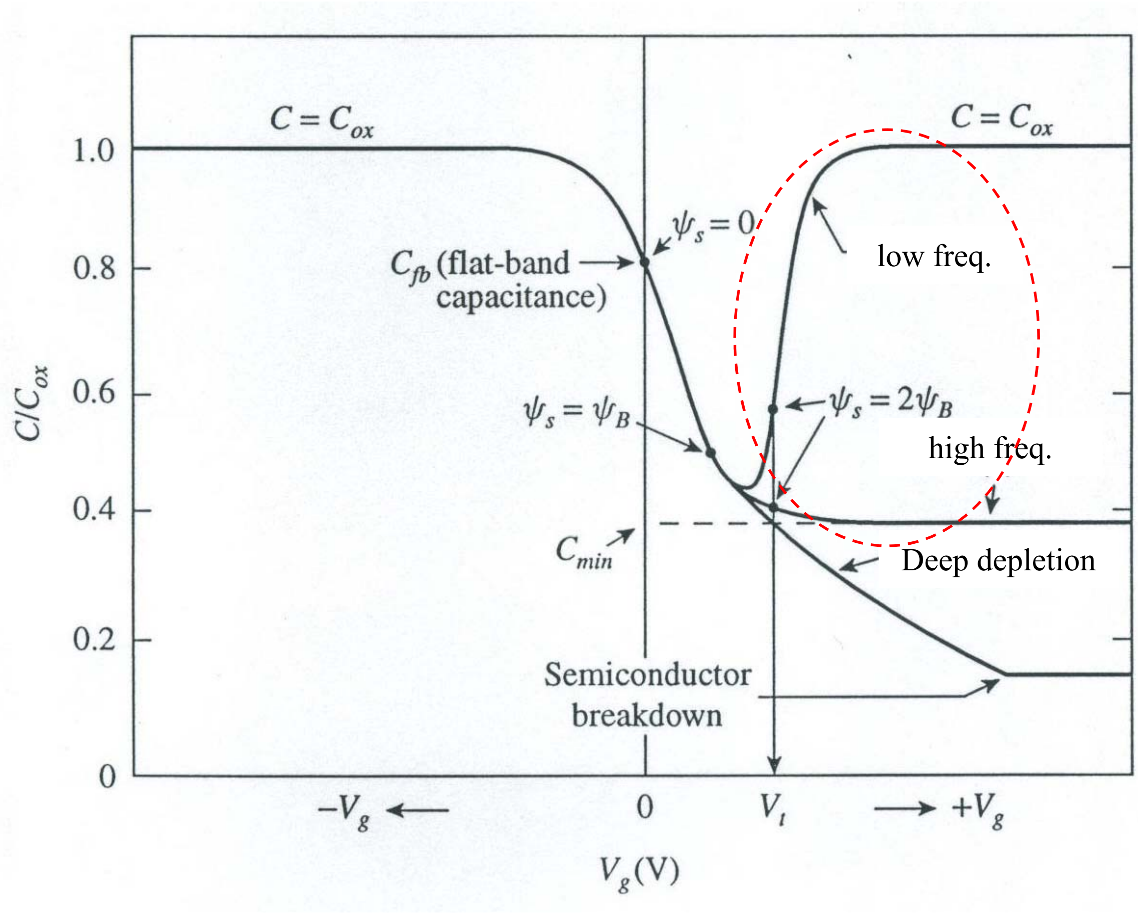
$$C_d = \frac{d(-Q_d)}{d\psi_s} = \sqrt{\frac{\varepsilon_{si} q N_a}{2\psi_s}} = \frac{\varepsilon_{si}}{W_d}$$

Note that

$$V_g = \frac{qN_a W_d}{C_{ox}} + \psi_s = \frac{\sqrt{2\varepsilon_{si} q N_a \psi_s}}{C_{ox}} + \psi_s$$

Solving the quadratic equation for ψ_s and will give C_d as a function of V_g

Capacitance-Voltage Characteristics – Inversion



- Inversion, high freq.: Inversion charge cannot respond,

$$\frac{1}{C_{\min}} = \frac{1}{C_{ox}} + \sqrt{\frac{4kT\ln(N_a/n_i)}{\varepsilon_{si}q^2N_a}}$$

- Inversion, low freq., or connected to a reservoir:

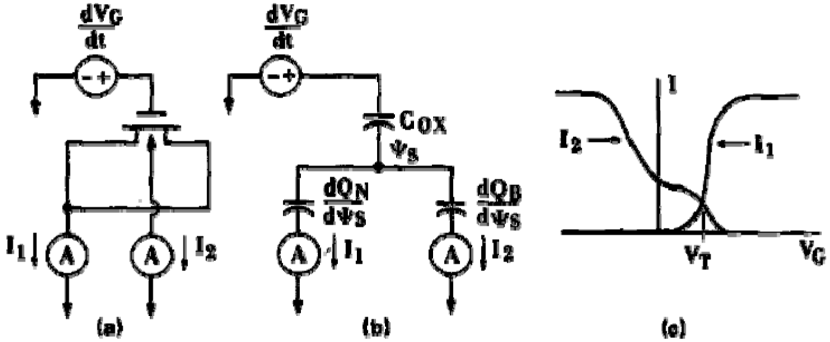
$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_d + C_i}$$

where

$$C_i = \frac{d(-Q_i)}{d\psi_s} = \frac{|Q_i|}{2kT/q}$$

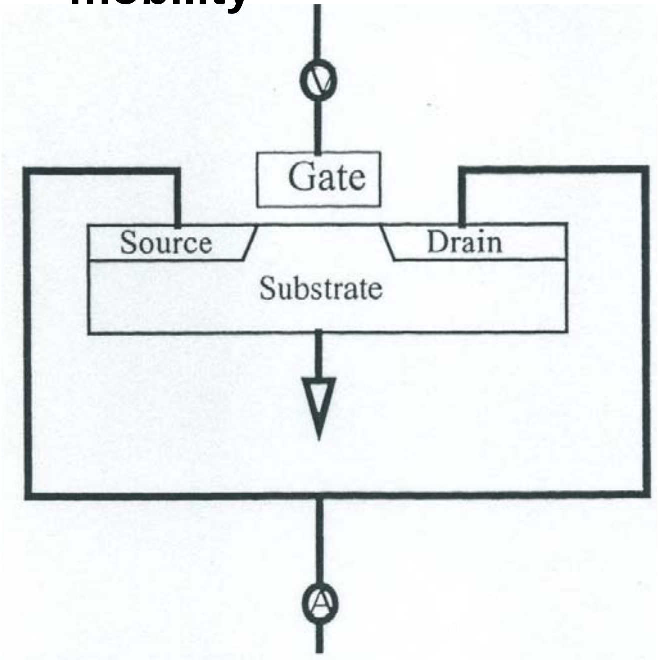
is the inv. layer cap.

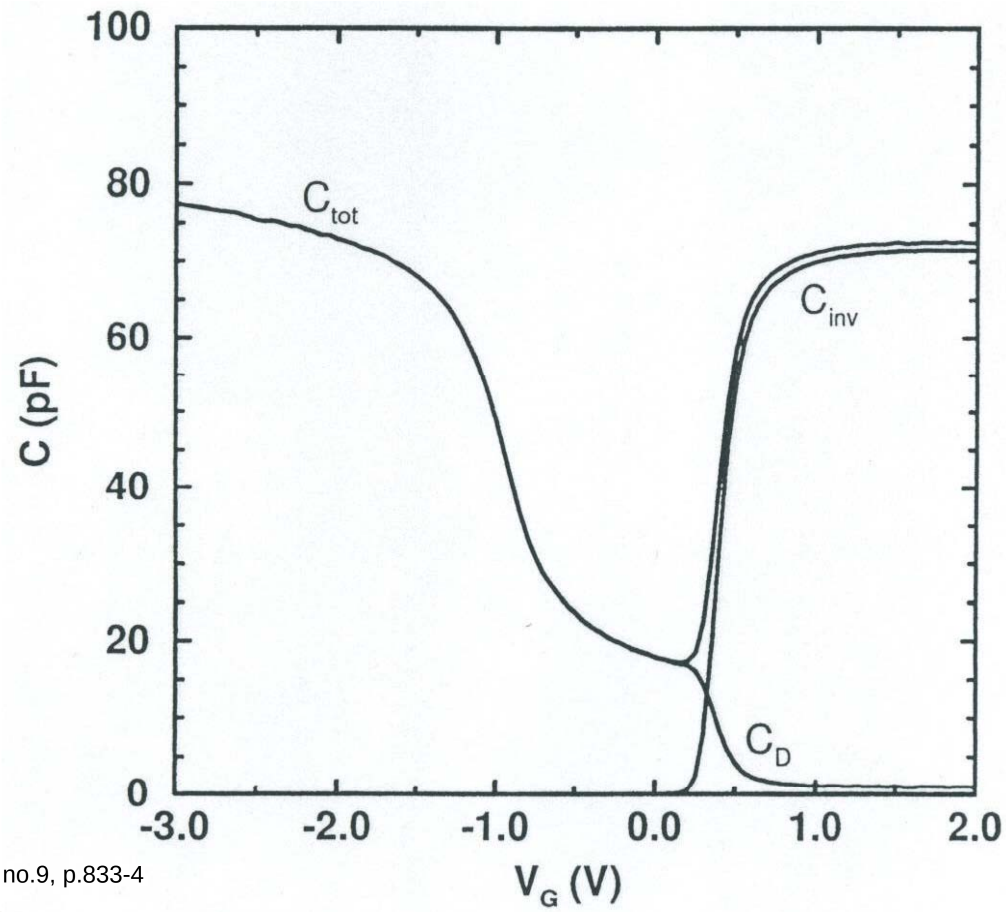
Like accumulation, $\frac{1}{C} = \frac{1}{C_{ox}}\left[1 + \frac{2kT/q}{|V_g - \psi_s|}\right]$ $(C_i >> C_d)$



Split C-V Measurement

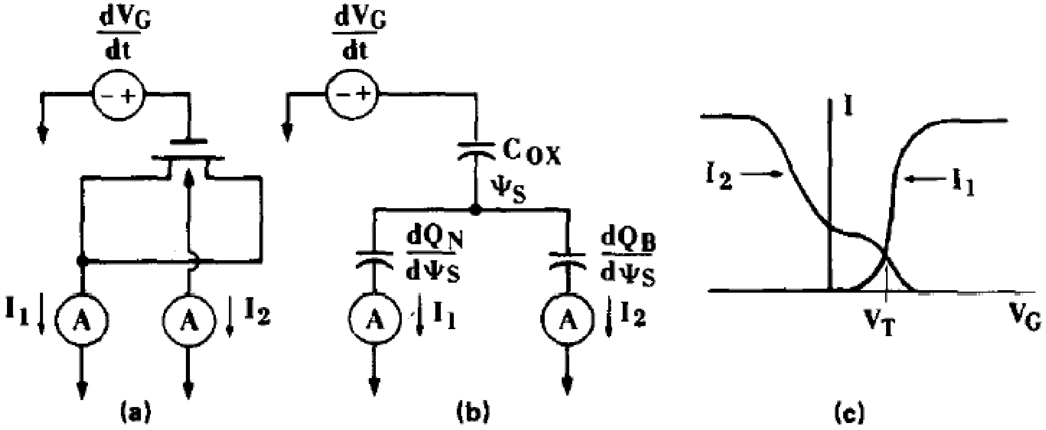
- **Measures the inversion charge (Q_{inv}) and depletion charge (Q_D) separately and directly**
- **Required to extract carrier mobility**





C. Sodini, T. Ekstedt, J. Moll, *Solid-State Electronics*; Sept. 1982; vol.25, no.9, p.833-4

Split C-V Basics



$$I_1 = \frac{dQ_N}{d\psi_S}\frac{d\psi_S}{dt}\cdots(1)$$

$$I_2 = \frac{dQ_B}{d\psi_S}\frac{d\psi_S}{dt}\cdots(2)$$

$$\psi_S = V_G - V_{FB} - \frac{Q_S}{C_{ox}}\cdots(3)$$

$$\frac{d\psi_S}{dV_G} = \frac{C_{ox}}{C_{ox} + \frac{dQ_S}{d\psi_S}}\cdots(4)$$

$$\frac{d\psi_S}{dt} = \frac{dV_G}{dt} - \frac{dQ_S}{dV_G}\frac{dV_G}{dt}\frac{1}{C_{ox}}\cdots(5)$$

$Q_S = Q_B + Q_N \cdots(6)$ use (4) and (6) get

$$\frac{dQ_S}{dV_G} = \frac{dQ_S}{d\psi_S}\frac{d\psi_S}{dV_G} = \frac{\frac{dQ_S}{d\psi_S}C_{ox}}{C_{ox} + \frac{dQ_N}{d\psi_S} + \frac{dQ_B}{d\psi_S}}\cdots(7)$$

$$\frac{dQ_B}{dV_G} = \frac{dQ_B}{d\psi_S}\frac{d\psi_S}{dV_G} = \frac{\frac{dQ_B}{d\psi_S}C_{ox}}{C_{ox} + \frac{dQ_N}{d\psi_S} + \frac{dQ_B}{d\psi_S}}\cdots(8)$$

$$\frac{dQ_N}{dV_G} = \frac{dQ_N}{d\psi_S}\frac{d\psi_S}{dV_G} = \frac{\frac{dQ_N}{d\psi_S}C_{ox}}{C_{ox} + \frac{dQ_N}{d\psi_S} + \frac{dQ_B}{d\psi_S}}\cdots(9)$$

Use (1), (5), and $(7)/(8)/(9)$, get

$$I_1 = \frac{dV_G}{dt}\left[\frac{\frac{dQ_N}{d\psi_S}C_{ox}}{C_{ox} + \frac{dQ_N}{d\psi_S} + \frac{dQ_B}{d\psi_S}}\right] = \frac{dV_G}{dt}\frac{dQ_N}{dV_G} = \alpha\frac{dQ_N}{dV_G}\cdots(10)$$

$$I_2 = \frac{dV_G}{dt}\left[\frac{\frac{dQ_B}{d\psi_S}C_{ox}}{C_{ox} + \frac{dQ_N}{d\psi_S} + \frac{dQ_B}{d\psi_S}}\right] = \frac{dV_G}{dt}\frac{dQ_B}{dV_G} = \alpha\frac{dQ_B}{dV_G}\cdots(10)$$

C. Sodini, T. Ekstedt, J. Moll, *Solid-State Electronics*; Sept. 1982; vol.25, no.9, p.833-4

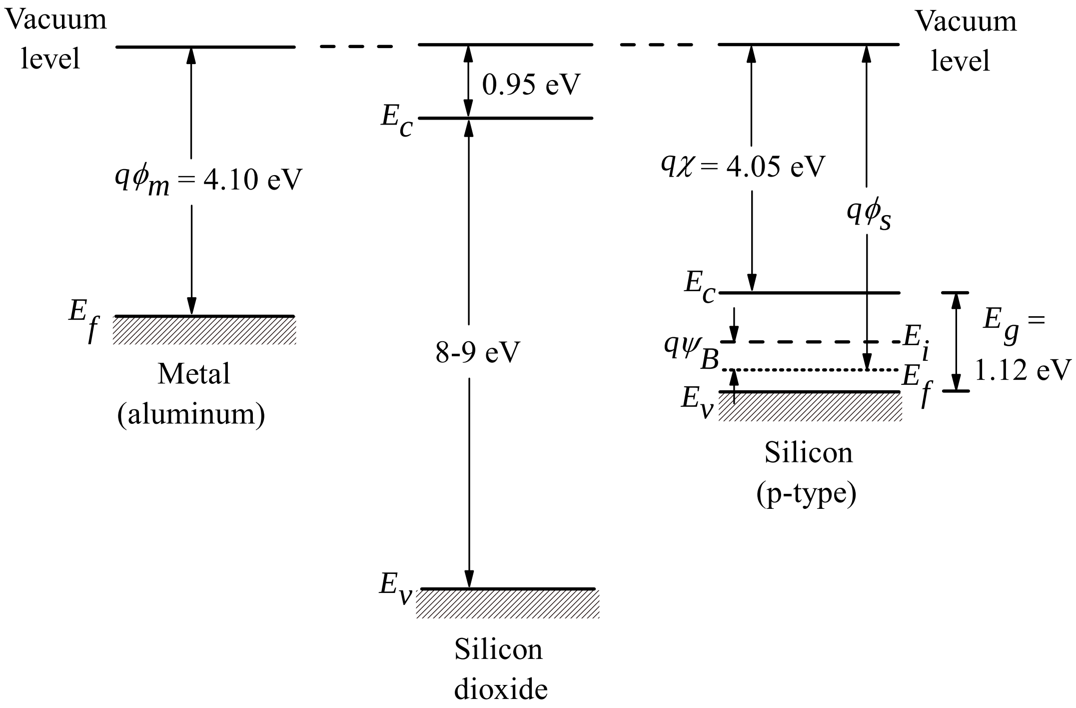
C-V Measurement

- **When performing CV measurements, start from inversion and sweep the gate voltage to accumulation**
- **Inversion layer build up is a slower process than collapsing the inversion layer and forming the accumulation layer (see Taur & Ning p. 28, Chapter 2.1.4.6)**

Flatband Voltage

Zero flatband voltage:

$$V_g = \psi_s + V_{ox} = \psi_s - \frac{Q_s}{C_{ox}}$$

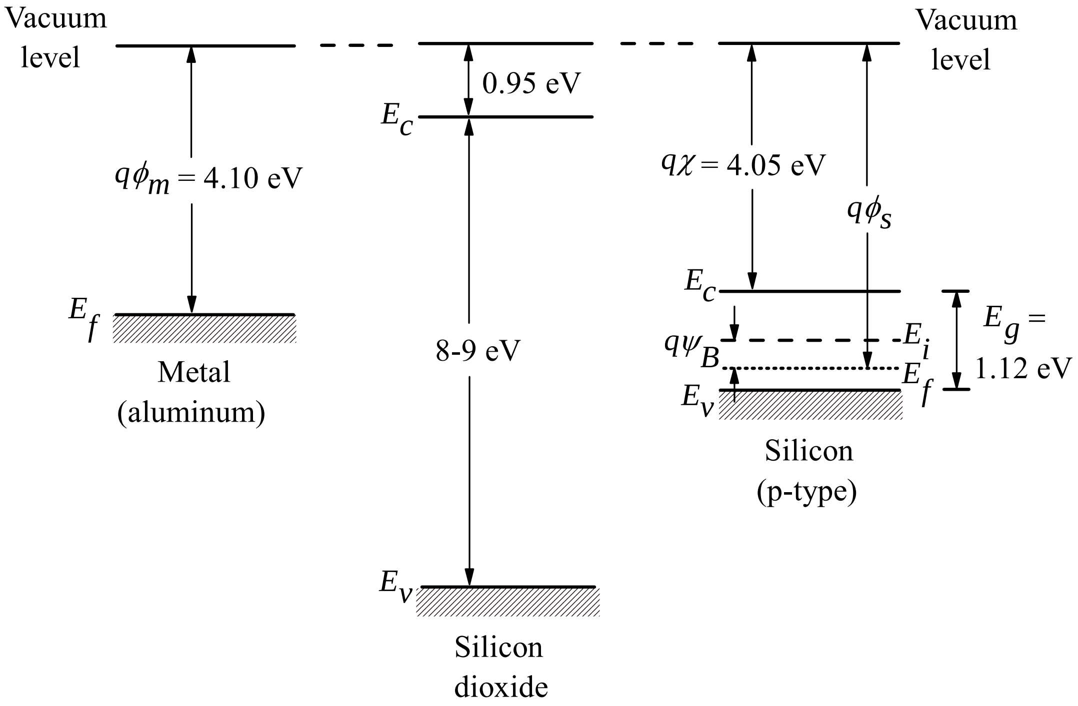


$$V_g = V_{FB} + \psi_s + V_{ox} = V_{FB} + \psi_s - \frac{Q_s}{C_{ox}}$$

Effect of Gate Work Function

$$V_t = V_{fb} + 2\psi_B + V_{ox} = V_{fb} + 2\psi_B + \frac{Q_d}{C_{ox}}$$

$$V_{fb} = (\phi_m - \phi_s) - \frac{Q_{ox}}{C_{ox}}$$



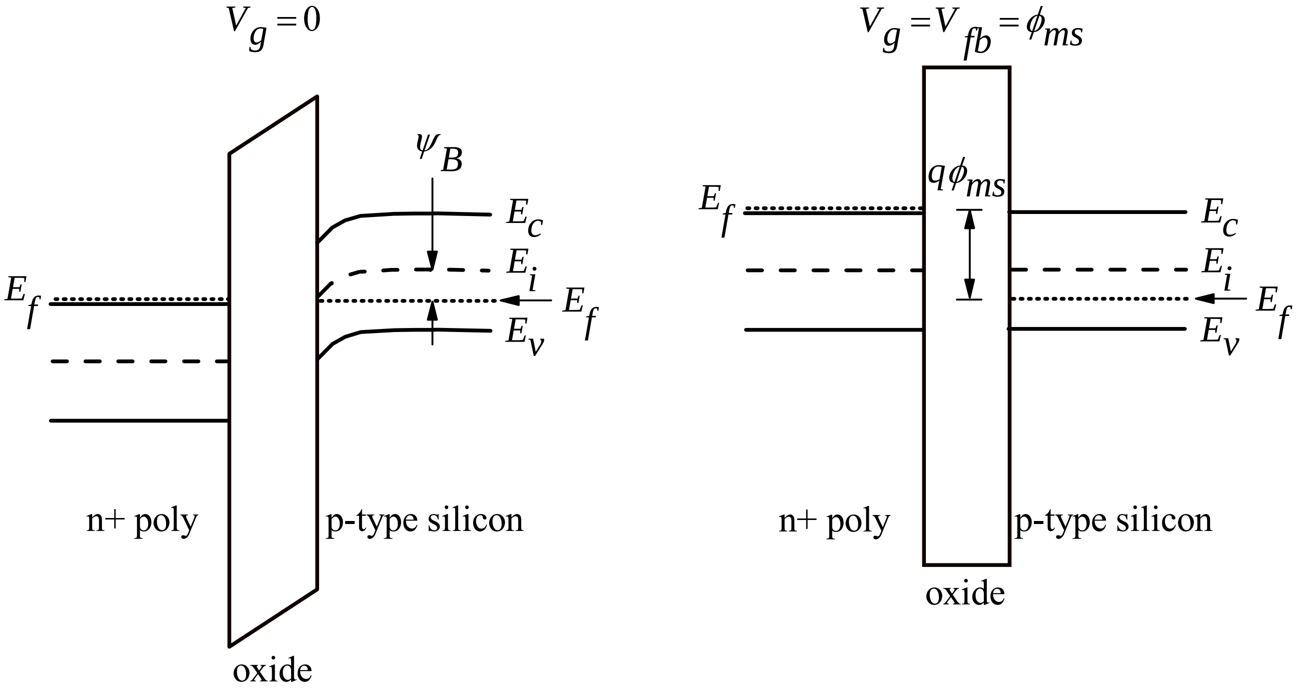
$$\phi_s = \chi + \frac{E_g}{2q} + \psi_B$$

$$\begin{cases} \phi_m = \chi \quad (\text{n}^+ \text{ poly}) \\ \phi_m = \chi + \dfrac{E_g}{2q} \quad (\text{midgap}) \\ \phi_m = \chi + \dfrac{E_g}{q} \quad (\text{p}^+ \text{ poly}) \end{cases}$$

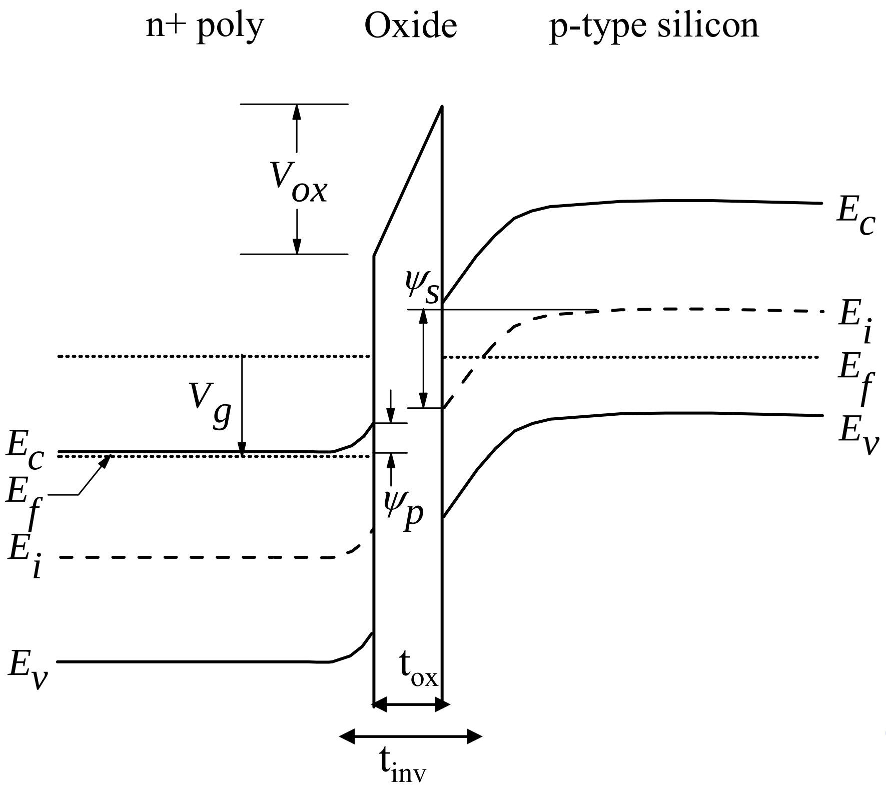
Effect of Gate Work Function

Example: n$^+$ polysilicon gate on p-type silicon

$$\phi_{ms} = -\frac{E_g}{2q} - \psi_B = -\ 0.56 - \frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right)$$



Polysilicon Gate Depletion Effect

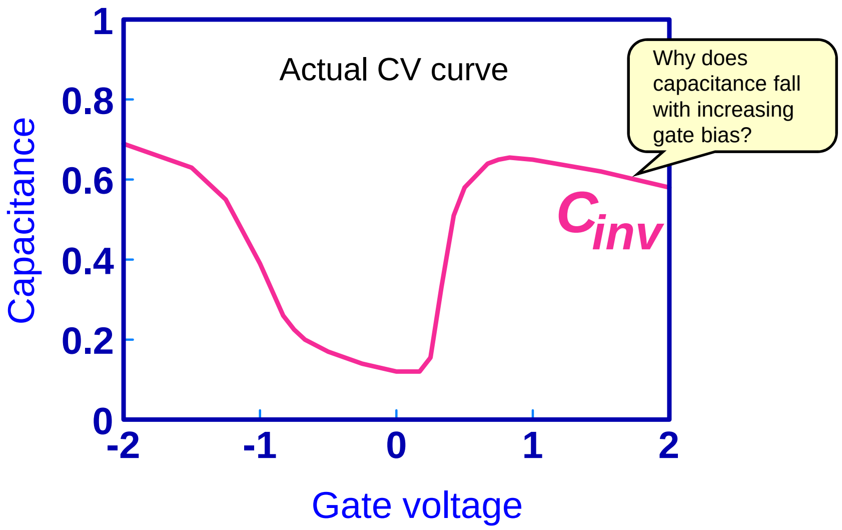


Gate eq. becomes:

$$V_g = V_{fb} + \psi_s + \psi_p - \frac{Q_s}{C_{ox}}$$

and,

$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_{si}} + \frac{1}{C_p}$$



Why does capacitance fall with increasing gate bias?

Typically, t_{inv} is 0.7-1.0 nm thicker than t_{ox}.