Homework #6 Solutions

1 (a) In VERY strong inversion, the $e^{q\psi_s/kT}$ term dominates in Equation 2.154 (4th term)

$$Q_I' \cong Q_s' \cong \sqrt{2K_s\epsilon_0 kT\,(n_i^2/N_d)}\ \exp\left(-q\psi_s/2kT\right)$$

With poly depletion, $\psi_{poly} = -\dfrac{Q_{poly}^2}{2K_s\epsilon_0 q N_{poly}} = -\dfrac{Q_s^2}{2K_s\epsilon_0 q N_{poly}}$

$$V_{gb} = \phi_{ms} + \psi_{poly} + \psi_{ox} + \psi_s \qquad \psi_{ox} = -\frac{Q_s'}{C_{ox}'}$$

$$\frac{dQ_s'}{dV_{gb}} = \frac{dQ_s'}{d\psi_s} \Big/ \frac{dV_{gb}}{d\psi_s}$$

$$\frac{dQ_s'}{d\psi_s} = -\frac{q}{2kT}Q_s' \qquad \frac{dV_{gb}}{d\psi_s} = 1 - \left[\frac{2Q_s}{2K_s\epsilon_0 q N_{poly}} + \frac{1}{C_{ox}'}\right]\frac{dQ_s}{d\psi_s}$$

$$\frac{dQ_s'}{dV_{gb}} = \frac{-\frac{q}{2kT}Q_s'}{1 + \frac{q}{2kT}Q_s'\left[\frac{1}{C_{ox}'} + \frac{Q_s'}{K_s\epsilon_0 q N_{poly}}\right]}$$

$$= -\frac{1}{\frac{2kT/q}{Q_s'} + \frac{1}{C_{ox}'} + \frac{Q_s'}{K_s\epsilon_0 q N_{poly}}}$$ see eq. 2.185 in text

$$C_{ox}' = 1.15\times10^{-6}\ \mathrm{F/cm^2}$$

(b) In inversion, $Q'_{I0} \cong -C'_{ox}\left(V_{GB}-V_T\right)$ without polydepletion or x_I

$$Q'_{I0} \cong \begin{cases} 3.45\times10^{-7}\ \text{C/cm}^2 & V_{gb} = V_T - 0.3\,V \\ 1.15\times10^{-6}\ \text{C/cm}^2 & = V_T - 1\,V \end{cases}$$

$$\frac{2kT/q}{Q'_S} \cong \begin{cases} 1.5\times10^{5}\ \text{cm}^2/\text{F} & V_T - 0.3\,V \\ 4.5\times10^{4}\ \text{cm}^2/\text{F} & V_T - 1\,V \end{cases}$$

$$\frac{Q'_S}{K_s\epsilon_0 q N_{poly}} = \begin{cases} 5.2\times10^{4}\ \text{cm}^2/\text{F} & V_T - 0.3\,V \\ 1.7\times10^{5}\ \text{cm}^2/\text{F} & V_T - 1\,V \end{cases}$$

$$1/C'_{ox} = 8.7\times10^{5}\ \text{cm}^2/\text{F}$$

For $V_{gb}-V_T=-1V$, poly depletion is most important. Inversion layer thickness is significant for lower overdrive (smaller Q'_s).

Due to x_{inv}

$$\frac{dQ'_s}{dV_{GB}} \text{ reduced by } \begin{cases} 0.85 = \dfrac{8.7\times10^5}{1.5\times10^5 + 8.7\times10^5} & V_T - 0.3 \\ 0.95 & V_T - 1\,V \end{cases}$$

Due to x_{poly}

$$\frac{dQ'_s}{V_{GB}} \text{ reduced by } \begin{cases} 0.94 & V_T - 0.3\,V \\ 0.84 & V_T - 1\,V \end{cases}$$

For both, $$\frac{dQ_s}{V_{GB}} \text{ reduced by } \begin{cases} 0.81 & V_T - 0.3\,V \\ 0.80 & V_T - 1\,V. \end{cases}$$

For the actual drop in total Q_s due to these effects, we need to solve

$$\psi_B = -\frac{kT}{q}\ln\left(\frac{N_d}{n_i}\right)$$

$$V_{gb} = \Phi_{ms} + \psi_{poly} + \psi_{ox} + \psi_s$$

$$= \Phi_{ms} + \frac{-Q_s'^2}{2K_s\epsilon_0 q N_{poly}} - \frac{Q_s'}{C_{ox}'} - \frac{2kT}{q}\ln\left[\frac{Q_s'}{\sqrt{2K_s\epsilon_0 kT(n_i^2/N_d)}}\right]$$

$$\cong V_T - 2\psi_B - \frac{Q_I'^2}{2K_s\epsilon_0 q N_{poly}} - \frac{Q_I'}{C_{ox}'} - \frac{2kT}{q}\ln\left[\quad\right]$$

$$V_{gb} - V_T = \frac{-Q_I'^2}{1.34\times10^{-11}\,\frac{CF}{cm^4}} - \frac{Q_I'}{1.15\times10^{-6}\,\frac{F}{cm^2}} - 0.052V\ln\left[\frac{Q_I'}{9.3\times10^{-8}\,\frac{C}{cm^2}}\right]$$

Including just poly depletion (first two terms), can solve quadratic

$$Q_I = \begin{cases} 3.35\times10^{-7} & 97\% & 0.3V \\ 1.06\times10^{-6} & 92\% & 1.0V \end{cases}$$

These are higher than change in $\frac{dQ}{dV}$ since average depth into poly is $x_d/2$

Including just $\bar{x}_I$

$$Q_I = \begin{cases} 2.76\times10^{-7} & 80\% & 0.3 \\ 1.02\times10^{-6} & 89\% & 1.0V \end{cases}$$

These are lower as initially charge is added deeper into silicon

Now $\bar{x}_I$ dominates in both cases.

2 (a) $Q'_{dm} = -\sqrt{2K_s\epsilon_0 q N_a (2\psi_B + V_{CB})}$ $V_{SB}=0$, so $V_{CB}=V_{CS}$

$$\cong -Q'_{dm}(V_{CB}=0) + \left.\frac{dQ'_{dm}}{dV_{CS}}\right|_{V_{CS}=0} V_{CS}$$

$$= -\sqrt{2K_s\epsilon_0 q N_a (2\psi_B)} + \frac{1}{2}\sqrt{2K_s\epsilon_0 q N_a}\,(2\psi_B)^{-1/2}\, V_{CS}$$

$$= -\xi\sqrt{2\psi_B} - \frac{\xi V_{CS}}{2\sqrt{2\psi_B}}$$

$$Q_I = -C'_{ox}\left[V_{GB} - \left(V_{FB} + 2\psi_B + V_{CS} + \frac{\xi\sqrt{2\psi_B}}{C'_{ox}} + \frac{\xi V_{CS}}{2C'_{ox}\sqrt{2\psi_B}}\right)\right]$$

$$= -C'_{ox}\left[V_{GS} - V_T - V_{CS} - \frac{\xi V_{CS}}{2C'_{ox}\sqrt{2\psi_B}}\right]$$

$$W\,dV = \frac{I_{DS}\,dy}{-Q_I \mu'_n} \Rightarrow \int_0^L I_{DS}\,dy = \int_0^{V_{DS}} -Q_I W \mu'_n\, dV$$

$$\cancel{L} I_{DS} = \mu'_n \frac{W}{L} C'_{ox}\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\left(1 + \frac{\xi}{2C'_{ox}\sqrt{2\psi_B}}\right)\right]$$

$$m-1 = \delta = \frac{\xi}{2C'_{ox}\sqrt{2\psi_B}} = \sqrt{\frac{\cancel{2}K_s\epsilon_0 q N_a}{\cancel{2}\psi_B}}\,\frac{1}{2C'_{ox}} = \frac{1}{C'_{ox}}\sqrt{\frac{K_s\epsilon_0 q N_a}{4\psi_B}}$$

as in Eq. 3.22

$C'_{ox} = 3.45\times10^{-7}\ F/cm^2$, $\xi = 3.17\times10^{-7}\ \frac{F V^{1/2}}{cm^2}$, $2\psi_B = 0.87V$

$\delta = 0.49$ (rather large for a modern device)

$$V_{fb} = \phi_{ms} - \frac{Q'_{ox}}{C'_{ox}} = -0.9V - \frac{3.2\times10^{-9}}{3.45\times10^{-7}} = -0.91V$$

(b) For $V_{GS} = V_{DS} = 3V$, transistor is in saturation

$V_{GD} < V_T$

In saturation:

$$I_{DS} = \mu_{eff} C_{ox}' \frac{W}{L} \frac{(V_{gs} - V_T)^2}{2m} \quad \text{(linearized model)}$$

$$I_{DS} = \mu_{eff} C_{ox}' \frac{W}{L}\left[\left(V_{gs} - V_{fb} - 2\psi_B - \frac{V_{ds}^{sat}}{2}\right) V_{ds}^{sat} - \frac{2\gamma}{3C_{ox}'}\left[(2\psi_B + V_{sb} + V_{ds}^{sat})^{3/2} - (2\psi_B + V_{sb})^{3/2}\right]\right]$$

where V_{ds}^{sat} can be found by setting $Q_c = -C_{ox}(V_{gs} - V_{fb} - 2\psi_B - V_{ds}^{sat}) + \sqrt{2K_s\epsilon_0 q N_a (2\psi_B + V_{sb} + V_{ds}^{sat})} = 0$

$$\gamma^2 (2\psi_B + V_{sb} + V_{ds}^{sat}) = C_{ox}'^2 (V_{gs} - V_{fb} - 2\psi_B - V_{ds}^{sat})^2$$

$$V_{ds}^{sat} = V_{gs} - V_{fb} - 2\psi_B + \frac{\gamma^2}{2C_{ox}'^2} - \sqrt{\left(V_{gs} - V_{fb} - 2\psi_B + \frac{\gamma^2}{2C_{ox}'^2}\right)^2 - (V_{gs} - V_{fb} - 2\psi_B)^2 + \frac{\gamma^2}{C_{ox}'^2}(2\psi_B + V_{sb})}$$

$$V_{ds}^{sat} = V_{gs} - V_{fb} - 2\psi_B + \frac{\gamma^2}{2C_{ox}'^2} - \sqrt{\frac{\gamma^2}{2C_{ox}'^2}\left(2(V_{gs} - V_{fb} + V_{sb}) + \frac{\gamma^2}{2C_{ox}'^2}\right)}$$

$$= 1.19\,V$$

$$m = \frac{(V_{gs} - V_T)^2}{2\left[\left(V_{gs} - V_{fb} - 2\psi_B - \frac{V_{ds}^{sat}}{2}\right) V_{ds}^{sat} - \frac{2\gamma}{3C_{ox}'}\left((2\psi_B + V_{sb} + V_{ds}^{sat})^{3/2} - (2\psi_B + V_{sb})^{3/2}\right)\right]}$$

$$V_T = V_{fb} + 2\psi_B + \frac{\gamma}{C_{ox}'}(2\psi_B + V_{sb})^{1/2} = 1.38\,V$$

$$m = \frac{(3 - 1.38)^2}{2\left[\left(3 + 0.91 - 0.87 - \frac{1.19}{2}\right)1.19 - \frac{2}{3}\left(\frac{3.17}{3.45}\right)\left[(0.87 + 2 + 1.19)^{3/2} - (0.87 + 2)^{3/2}\right]\right]}$$

$= 1.50 \quad (\gamma = 0.5)$

For $V_{DS} = 1.5V$, still in saturation, so $\delta = 0.5$ gives correct current

The error due to $\delta = 0.49$ is a factor of $\frac{m_1}{m_2} = \frac{1.5}{1.49} = 1.01$

or less than 1%.

Note that the agreement is particularly good for this example since ψ_s at source is particularly large due to body bias ~~and~~ (2.87V) and doesn't increase that much at pinch-off due to relatively high V_T. Thus the dependence of charge in depletion region on V_{GS} is nearly linear as assumed.