EE531
Winter 2007

Homework #1 Solutions

1. $$U = \frac{pn - n_i^2}{\tau_p[n+n_t] + \tau_n[p+p_t]}$$

$n_t = n_i \exp\left(\frac{E_t - E_i}{kT}\right)$

$p_t = n_i \exp\left(\frac{E_i - E_t}{kT}\right)$

$\tau_p = (\sigma_p N_t v_{thp}) = \begin{cases} 1.67\times10^{-5}\ \text{s} \\ 1.67\times10^{-7}\ \text{s} \end{cases}$ $\quad E_t = E_f$, $\quad E_t = E_f + 0.2\text{eV}$

$v_{thp} = 6\times10^6$ cm/s

$\tau_{n1} = 10^{-5}$ s ; $\tau_{n2} = 10^{-7}$ s ; $n_{t1} = p_t = n_i = 10^{10}$ cm^{-3}

$n_{t2} = n_i \exp\left(\frac{0.2}{0.026}\right) = 2.2\times10^{13}$ cm^{-3}

$U = U_1 + U_2$ $\qquad p_{t2} \ll n_i$

$n_0 = n_i$

$$U_1 \cong \frac{pn - n_i^2}{\tau_p[n_i+n_i] + \tau_n(n_i+n_i)}$$

$$\cong \frac{(n_i+\Delta p)(n_i+\Delta p) - n_i^2}{2n_i(\tau_{p1}+\tau_{n1})} = \frac{\cancel{2n_i}\Delta p + \underbrace{\cancel{\Delta p^2}}_{\ll n_i\Delta p}}{\cancel{2n_i}(\tau_{p1}+\tau_{n1})} = \frac{\Delta p}{\tau_{p1}+\tau_{p2}}$$

$$U_2 \cong \frac{(n_i+\Delta p)(n_i+\Delta p) - n_i^2}{\tau_{p2}[n_i + 2200n_i] + \tau_{n2}[n_i]} \cong \frac{2\cancel{n_i}\Delta p}{2200\cancel{n_i}\tau_{p2}} = \frac{\Delta p}{1100\tau_{p2}}$$

$\tau_{p1} + \tau_{p2} = 2.67\times10^{-5}$ s ; $1100\tau_{p2} = 1.84\times10^{-4}$ s

$U_1 \gg U_2 \Rightarrow$ centers at midgap dominate

$$\tau_{eff} = \frac{1}{\frac{1}{1100\tau_{p2}} + \frac{1}{\tau_{p1}+\tau_{n1}}} = \underline{\underline{2.33\times10^{-5}\ \text{s}}}$$

$n_0 = 10^{17}$ cm^{-3} $\quad U_1 \cong \frac{n_0\Delta p}{\tau_{p1}n_0} = \frac{\Delta p}{\tau_{p1}}$; $U_2 \cong \frac{n_0\Delta p}{\tau_{p2}n_0} = \frac{\Delta p}{\tau_{p2}}$ $\qquad \tau_{eff} = \frac{\tau_{p1}\tau_{p2}}{\tau_{p1}+\tau_{p2}} = \underline{1.65\times10^{-7}\ \text{s}}$

$\tau_{p2} \ll \tau_{p1} \Rightarrow$ centers above midgap dominate

(b) For $n_0 >> 10^{18}$ Auger dominates

$$\tau_p = \frac{1}{K_n n_0^2} \quad (\text{l.l.i.}) \qquad \tau_p(3\times10^{19}) \cong 10^{-8}\,s$$

$$\text{Thus } K_n = \frac{1}{\tau_p n_0^2} = \frac{1}{10^{-8}s\,(3\times10^{19}cm^{-3})^2} = 1.1\times10^{-31}\,cm^6 s^{-1}$$

(c) For $n > n_0 = 10^{17}\,cm^{-3}$, $U_2 >> U_1$; $U = U_2 + U_{Auger}$

For $\Delta p << n_0 = 10^{17}$ $\quad \tau \cong \tau_{p2} = 1.67\times10^{-7}s$ (as in (a))

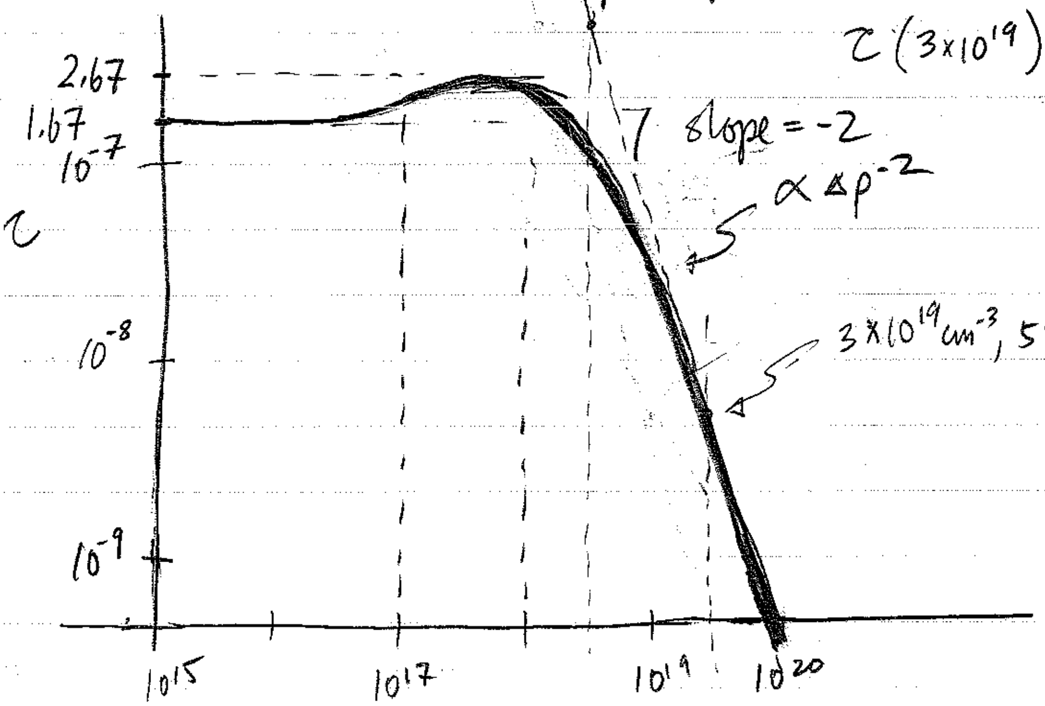
For $n_0 << \Delta p << 10^{18}$ $\qquad n \cong p \cong \Delta p$ and $\tau \cong \tau_{p2} + \tau_{n2}$

$$U \cong U_2 \cong \frac{\Delta p^2}{\tau_{p2}(\Delta p) + \tau_{n2}(\Delta p)} \qquad \cong 2.67\times10^{-7}s$$

For $\Delta p >> 10^{18}$ $\quad U \cong U_{Auger} = (K_n \Delta p^2 + K_p \Delta p^2)\Delta p$

$n \cong p \cong \Delta p$ $\qquad \tau \cong \dfrac{1}{(K_n + K_p)\Delta p^2}$ $\quad$ Assume $K_p \cong K_n$

$\tau(3\times10^{19}) \cong 5\times10^{-9}s$



2 (a) $L_p = \sqrt{D_p \tau_p} = 6.3\,\mu m \sim W = 5\,\mu m$

Assume low-level injection, charge neutrality, diffusion approximation

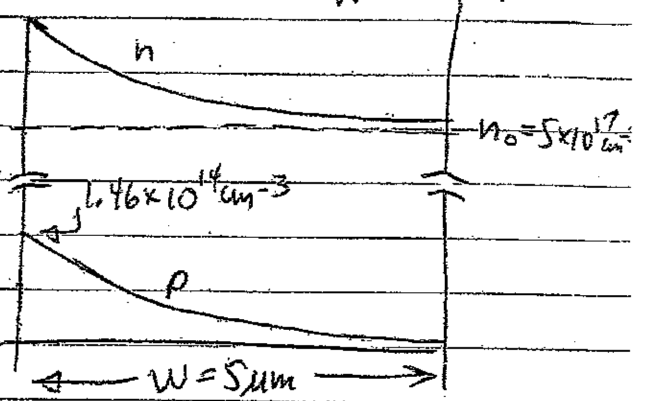
$n = n_0 + \Delta n \cong n_0$

$p = p_0 + \Delta p \cong \Delta p$

$$\frac{d\Delta p}{dt} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p}$$

$\underset{\text{in s.s}}{0} =$

$$\Delta p = A e^{-x/L_p} + B e^{x/L_p}$$



BCs $x=0$ $G_{L_s} = J_p(0)/q$ $x=W$ $\frac{1}{q}J_p(W) = U_s$

$10^{18}\,cm^{-2}s^{-1} = -D_p \frac{d\Delta p}{dx}\Big|_{x=0}$ $-D_p \frac{d\Delta p}{dx}\Big|_{x=W} = S\Delta p(W)$

$10^{18}\,cm^{-2}s^{-1} = \frac{D_p}{L_p}(A - B)$ $\frac{D_p}{SL_p}\left(Ae^{-W/L_p} - Be^{+W/L_p}\right) = \left(Ae^{-W/L_p} + Be^{W/L_p}\right)$

$A - B = 1.6\times10^{14}\,cm^{-3}$ $0.63\left(\frac{A}{2.2} - 2.2B\right) = \frac{A}{2.2} + 2.2B$

$A = -21.3B$ $B = -7.2\times10^{12}\,cm^{-3}$, $A = 1.53\times10^{14}\,cm^{-3}$

(b) $U_s = S\Delta p(W) = 10^4 \frac{cm}{s}\left[\frac{1.53}{2.2} - 0.07 * 2.2\right] * 10^{14}\,cm^{-3} = 5.4\times10^{17}\,cm^{-2}s^{-1}$

$\frac{U_s}{G_{L_s}} = 0.54$ ⇒ 54% recombine at W = 5 µm
46% recombine in bulk

(c) $\Delta n \cong \Delta p$, $D_n \cong 10\ cm^2/s$ (from graph or $2.5 * D_p$)

$$J_n^{diff} = qD_n \frac{d(\Delta n)}{dx} = qD_n\left[-\frac{A}{L_p}e^{-x/L_p} + \frac{B}{L_p}e^{x/L_p}\right]$$

$$= (1.6\times10^{-19}C)\left(10\frac{cm^2}{s}\right)\left[-\frac{1.53\times10^{14}cm^{-3}e^{-x/L_p}}{6.3\times10^{-4}cm} + \frac{7.2\times10^{12}cm^{-3}e^{x/L_p}}{6.3\times10^{-4}cm}\right]$$

$$= -\left[0.39e^{-x/6.3\mu m} + 0.02e^{x/6.3\mu m}\right]A/cm^2$$

(d) $J_p \cong J_p^{diff} = -\frac{D_p}{D_n}J_n^{diff}$ since $\Delta n \cong \Delta p$

but $J_p = -J_n$ (same fluxes, opposite currents)

$$= -\left(J_n^{diff} + J_n^{drift}\right)$$

$$J_n^{drift} = \left[\frac{D_p}{D_n} - 1\right]J_n^{diff} = -0.6J_n^{diff} = -\left[0.23e^{-x/6.3\mu m} + 0.01e^{x/6.3\mu m}\right]$$

$$= q\mu_n n\mathcal{E} = (1.6\times10^{-19}C)\left(\frac{10\,cm^2/s}{0.026V}\right)(5\times10^{17}cm^{-3})\,\mathcal{E}$$

$$\mathcal{E} = \frac{J_n^{drift}}{30.8\,(\Omega\text{-}cm)^{-1}} = -\left[7.5\times10^{-3}e^{-x/6.3\mu m} + 3.5\times10^{-4}e^{x/6.3\mu m}\right]$$

(e) $J_p^{drift} = q\mu_p p\,\mathcal{E} = J_n^{drift}\left(\frac{\mu_p}{\mu_n}\right)\left(\frac{p}{n}\right) =$

$$= -\frac{\left(1.53\times10^{14}cm^{-3}e^{-x/L_p} + 7.2\times10^{12}cm^{-3}e^{x/L_p}\right)}{(2.5)(5\times10^{18}cm^{-3})}\left(0.23e^{-x/L_p} + 0.01e^{x/L_p}\right)$$

$$\frac{J_p^{drift}}{J_p^{diff}} = \frac{J_n^{drift}(\mu_p/\mu_n)(p/n)}{-(D_p/D_n)J_n^{diff}} = \frac{-0.6J_n^{diff}(p/n)}{-J_n^{diff}} = 0.6\frac{p}{n} < 1.8\times10^{-5}$$

so Diffusion Approximation is excellent

(f) $$\frac{\rho}{q} = \frac{K\epsilon_0}{q}\frac{d\mathcal{E}}{dx} = \frac{12\left(8.854\times10^{-14}\frac{F}{cm}\right)}{(1.6\times10^{-19}C)}\left[\frac{7.5\times10^{-3}V/cm\,e^{-x/L_p}}{6.3\times10^{-4}cm} - \frac{3.5\times10^{-4}V/cm}{6.3\times10^{-4}cm}e^{x/L_p}\right]$$

$$= 7.9\times10^{10}e^{-x/L_p} - 3.7\times10^{9}e^{x/L_p} \ll \Delta p, \Delta n$$

Again, an excellent approx.