Homework #3 - EE 531

due 4/21/17

1. Consider silicon heavily doped with arsenic at a concentration of 4×10^{19}cm^{-3} such that an impurity band with density of states

$$N_{ib}(E) = \begin{cases} \dfrac{4\pi \times 10^{19}\text{cm}^{-3}}{0.12eV} \cos\left[\dfrac{\pi(E - E_c + 0.03eV)}{0.12eV}\right] & E_c + 0.03eV > E > E_c - 0.09eV \\ 0 & \text{otherwise} \end{cases}$$

is formed. Assume that the conduction and valence band density of states, as well as $E_g = E_c - E_v$ are unchanged.

(a) If the electron concentration is equal to the donor doping, what would be the Fermi level location at 0K (absolute zero)? (Hint: Note simple form of Fermi Dirac statistics at 0K.)

(b) If the Fermi level remains in the same location as the temperature rises (as it will approximately in degenerate material), what would be the hole concentration at room temperature (300K)? How does the *pn* product (n_{ie}^2) compare to n_i^2 in lightly doped material (Assume all electrons in overlapping impurity/conduction band are mobile)?

2. A MOS capacitor is made with a silicon substrate doped with $N_a = 5{\times}10^{17}$cm^{-3} of boron, 6nm of silicon dioxide, and an n^+ polysilicon gate doped such that $E_f - E_c = 0.05$eV. $Q'_{ss}/q = 5 \times 10^{10}$ cm^{-2}. Assuming that inversion and accumulation charges approximate a sheet of charge at the interface and that weak inversion charges can be neglected, determine the charge on the gate, the voltage dropped across the oxide and the voltage dropped across the silicon with the following voltages applied between the gate and the substrate:

   (a) $V_{gb} = -1$V

   (b) $V_{gb} = 0.0$V

   (c) $V_{gb} = 2$V

   Sketch the charge densities, electric fields and energy band diagrams in each case. What are the capacitances at low and high frequencies in each of the above cases?

3. A reasonable approximate solution for the ground-state wavefunction of an electron in a triangular potential well with $V(x) = q\,\text{E}\,x$ for $x > 0$ (infinite for $x < 0$) is $C\,x\,\exp(-\gamma x/2)$. The variational principle says that optimum choice for γ is the value that minimizes expected value of energy (E).

   (a) As function of electric field E, what is the ground state energy and associated value of γ.

   (b) For Si, what is the expected value of the distance of electrons from the surface? Consider lowest sub-band for all six minima, plus the next lowest sub-band(s). Compare this to the classical solution (assume constant electric field). What is the equivalent effective oxide thickness (with same capacitance?

   (c) Considering the same set of sub-bands, plot the density of states versus energy. Comment on comparison to 3D density of states.

4. Consider an MOS capacitor with $x_{ox} = 3$ nm, a p^+ poly gate doped with $N_a = 4 \times 10^{19}$ cm^{-3}, a substrate uniformly doped with $N_d = 10^{18}$ cm^{-3} and negligible oxide charges.

   (a) Including poly depletion (use depletion approximation) and the finite thickness of the inversion layer (use appropriately simplified version of Eq. 2.154), derive an expression for $dQ_I^0/dV_{GB} \approx dQ_s'/dV_{GB}$ in very strong inversion ($Q_I >> Q_d$).

   (b) Using this expression, determine by what factor the inversion charge is reduced due to these two effects for $V_{GB} = V_T - 0.3V$ and $V_{GB} = V_T - 1V$, where V_T is defined by $\psi_s = 2\psi_B$. Which effect is more significant in each case?