EE 531
Spring 2009

Homework #4 Solutions

1 (a)

$V_{SB}=0$, so $V_{CB}=V_{CS}$

$$Q'_{dm} = -\sqrt{2K_s\epsilon_0 q N_a (2\psi_B + V_{CB})}$$

$$\cong -Q'_{dm}(V_{CB}=0) + \left.\frac{dQ'_{dm}}{dV_{CS}}\right|_{V_{CS}=0} V_{CS}$$

$$= -\sqrt{2K_s\epsilon_0 q N_a (2\psi_B)} + \frac{1}{2}\sqrt{2K_s\epsilon_0 q N_a}\,(2\psi_B)^{-1/2} V_{CS}$$

$$= -\xi\sqrt{2\psi_B} - \frac{\xi V_{CS}}{2\sqrt{2\psi_B}}$$

$$Q_I = -C'_{ox}\left[V_{GB} - \left(V_{FB} + 2\psi_B + V_{CS} + \frac{\xi\sqrt{2\psi_B}}{C'_{ox}} + \frac{\xi V_{CS}}{2C'_{ox}\sqrt{2\psi_B}}\right)\right]$$

$$= -C'_{ox}\left[V_{GS} - V_T - V_{CS} - \frac{\xi V_{CS}}{2C'_{ox}\sqrt{2\psi_B}}\right]$$

$$W\,dV = \frac{I_{DS}\,dy}{-Q_I \mu'_n} \Rightarrow \int_0^L I_{DS}\,dy = \int_0^{V_{DS}} (-Q_I) W \mu'_n\,dV$$

$$\frac{1}{\cancel{L}}\cancel{L}\, I_{DS} = \mu'_n \frac{W}{L} C'_{ox}\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\left(1 + \frac{\xi}{2C'_{ox}\sqrt{2\psi_B}}\right)\right]$$

$$m - 1 = \delta = \frac{\xi}{2C'_{ox}\sqrt{2\psi_B}} = \sqrt{\frac{\cancel{2}K_s\epsilon_0 q N_a}{\cancel{2}\psi_B}}\,\frac{1}{2C'_{ox}} = \frac{1}{C'_{ox}}\sqrt{\frac{K_s\epsilon_0 q N_a}{4\psi_B}}$$

as in Eq. 3.22

$C'_{ox} = 2.3\times10^{-6}\ \mathrm{F/cm^2}$, $\xi = 1.29\times10^{-6}\ \frac{\mathrm{F\,V^{1/2}}}{\mathrm{cm^2}}$, $2\psi_B = 1.04\ \mathrm{V}$

$\delta = 0.275$ for $V_{SB}=0$, $\delta = 0.196$ for $V_{SB} = 1\ \mathrm{V}$ $\left(\sqrt{2\psi_B} \Rightarrow \sqrt{2\psi_B + V_{SB}}\right)$

$$V_{fb} = \phi_{ms} - \frac{Q'_{ox}}{C'_{ox}} = -0.95\,\mathrm{V} - \frac{8\times10^{-9}}{2.3\times10^{-6}} = -0.953\,\mathrm{V}$$

(b) For $V_{GS} = V_{DS} = 1.5V$, transistor is in saturation
$V_{GD} < V_T$

In saturation:

$$I_{DS} = \mu_{eff} C_{ox}' \frac{W}{L} \frac{(V_{gs} - V_T)^2}{2m} \quad \text{(linearized model)}$$

$$I_{DS} = \mu_{eff} C_{ox}' \frac{W}{L}\left[\left(V_{gs} - V_{fb} - 2\psi_B - \frac{V_{ds}^{sat}}{2}\right) V_{ds}^{sat} - \frac{2\gamma}{3C_{ox}'}\left[(2\psi_B + V_{sb} + V_{ds}^{sat})^{3/2} - (2\psi_B + V_{sb})^{3/2}\right]\right]$$

where V_{ds}^{sat} can be found by setting $Q_c = -C_{ox}'(V_{gs} - V_{fb} - 2\psi_B - V_{ds}^{sat}) + \sqrt{2K_s\epsilon_0 q N_a (2\psi_B + V_{sb} + V_{ds}^{sat})} = 0$

$$\gamma^2 (2\psi_B + V_{sb} + V_{ds}^{sat}) = C_{ox}'^2 (V_{gs} - V_{fb} - 2\psi_B - V_{ds}^{sat})^2$$

Solving quadratic:

$$V_{ds}^{sat} = V_{gs} - V_{fb} - 2\psi_B + \frac{\gamma^2}{2C_{ox}'^2} \mp \sqrt{\frac{\gamma^2}{2C_{ox}'^2}\left((V_{gs} - V_{fb} + V_{sb})2 + \frac{\gamma^2}{2C_{ox}'^2}\right)}$$

$$= 0.54\ V$$

$$m = \frac{(V_{gs} - V_T)^2}{2\left[\left(V_{gs} - V_{fb} - 2\psi_B - \frac{V_{ds}^{sat}}{2}\right) V_{ds}^{sat} - \frac{2\gamma}{3C_{ox}'}\left((2\psi_B + V_{sb} + V_{ds}^{sat})^{3/2} - (2\psi_B + V_{sb})^{3/2}\right)\right]}$$

$$V_T = V_{fb} + 2\psi_B + \frac{\gamma}{C_{ox}'}(2\psi_B + V_{sb})^{1/2} = 0.89\,V$$

$$m = \frac{(1.5 - 0.89)^2}{2\left[\left(1.5 + 0.953 - 1.04 - \frac{0.54}{2}\right)0.54 - \frac{2}{3}\left(\frac{1.29}{2.3}\right)\left[(1.04 + 1 + 0.54)^{3/2} - (1.04 + 1)^{3/2}\right]\right]}$$

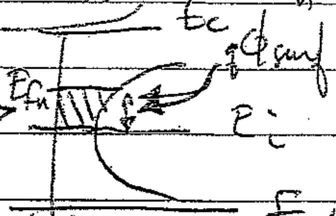
$= 1.184$ ($\delta = 0.184$) about 1.1% lower

$V_{ds} = 0.8$ also in saturation, so no error for $m = 1.184$ and 1.1% error (low) for $m = 1.196$

2. $q\phi^n_{surf} \equiv (E_{f_n} - E_i)_{surface} = q(\phi_F + \psi_s - V_{CB})$

(a) $Q'_{ss} = -\int_{E_i}^{q\phi^n_{surf}+E_i} q N'_{ss}(E)\, dE$

as E_{f_n} moves up in gap, more surface states are filled



$Q'_{ss} < 0$ when $\phi^n_{surf} > 0$

$Q'_{ss} > 0$ when $\phi^n_{surf} < 0$

$$Q'_{ss} = -\int_{E_i}^{q(\phi_F+\psi_s-V_{CB})+E_i} q N'_{ss}(E)\, dE \qquad -\phi_F = |\psi_B| = \frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) = 0.46V$$

$$= -q\left[5\times10^{10} E + \frac{10^{11}}{3(eV)^2}(E)^3\right]_0^{q(-0.46V+\psi_s-V_{CB})} cm^{-2}\,eV^{-1}$$

$$= -q\left[5\times10^{10}(-0.46V+\psi_s-V_{CB})\,V^{-1}cm^{-2} + \frac{10^{11}}{3}(-0.46V+\psi_s-V_{CB})^3\,V^{-3}cm^{-2}\right]$$

(b) $$C'_{it} = \left|\frac{dQ'_{ss}}{d\psi_s}\right| = +q\left[5\times10^{10}\,V^{-1}cm^{-2} + 10^{11}(-0.46V+\psi_s-V_{CB})^2\,V^{-3}cm^{-2}\right]$$

(c) $$V_{GB} = \phi_{ms} + \psi_s - \frac{Q'_{ss}}{C'_{ox}} + \gamma\sqrt{\psi_s} \qquad \phi_{ms} = -1.0\ V$$

$$C'_{ox} = 6.9\times10^{-7}\ F/cm^2$$

$$= -1V + \psi_s - \frac{Q'_{ss}}{C'_{ox}} + 0.59V^{1/2}\sqrt{\psi_s} \qquad \gamma = \frac{\sqrt{2K_s\epsilon_0 q N_a}}{C'_{ox}} = 0.59\,V^{1/2}$$

$$V_{GB}(\psi_s = V_{CB} + 2|\psi_F|) = -1V + V_{CB} + 2(0.46V) + \frac{q}{C'_{ox}}\left[5\times10^{10}(0.46V) + \frac{10^{11}}{3}(0.46V)^3V^{-2}\right]V^{-1}cm^{-2} + 0.59V^{1/2}\sqrt{V_{CB}+0.92V}$$

$$= -0.07V + 0.59V^{1/2}\sqrt{V_{CB}+0.92V} + V_{CB}$$

$$\frac{dV_{GB}}{d\psi_s} = 1 + \frac{C'_{it}}{C'_{ox}} + \frac{C'_d}{C'_{ox}} = \qquad \frac{C'_d}{C'_{ox}} = \frac{dQ_d}{d\psi_s} = \frac{1}{2}\gamma(\psi_s)^{-1/2} = 0.295V^{1/2}(0.92V+V_{CB})^{-1/2}$$

(d) w/o ss $m(V_{CB}=0) = 1.306$ w/ ss $m(V_{CB}=0) = 1.322$ $\left[\frac{C'_{it}}{C'_{ox}} = 0.016\right]$

$m(V_{CB}=V) = 1.149$ $m(V_{CB}=V) = 1.165$