Homework #6 - EE 531

due 5/29/09

1. Consider a planar symmetric dual gate Si NMOSFET with a thin fully-depleted body.
   (a) Derive an expression for the threshold voltage as function of the equivalent oxide thicknesses (equal), metal gate workfunctions (equal), semiconductor doping (uniform) and semiconductor thickness. Assume that both gate voltages are tied to the same input.
   (b) Evaluate your result for $t_{ox} = 1\,\text{nm}$, $x_s = 5\,\text{nm}$, $N_a = 2 \times 10^{19}\text{cm}^{-3}$ and n^+ poly gates ($E_f = E_c$). Sketch the band edges (e.g., E_c, E_v, E_F, etc.) versus distance from gate to gate for a cut midway between source and drain at the edge of strong inversion for $V_{ds} = 0$.

2. Consider a spatially uniform system with no generation/recombination. Using the hydrodynamic equations (see notes):
   (a) Determine an expression for the dependence of the average electron energy on electric field in steady-state if it is given that the momentum relaxation time depends inversely on the energy as $\tau_m = AE^{-3/2}$ and the steady-state drift velocity depends on the electric field as $|v_d| = v_{\text{sat}}\mathcal{E}/(\mathcal{E} + \mathcal{E}_c)$, where $\mathcal{E}$ is the electric field.
   (b) What must be the dependence of the energy relaxation time (τ_E) on average electron energy? Sketch your result.

3. The scattering rate due to ionized impurity scattering in a single parabolic and spherically symmetric band is:

$$S(k, k') = \frac{2\pi}{\hbar}\frac{N_I Z^2 q^4}{\Omega \epsilon_s^2}\frac{\delta(E_{k'} - E_k)}{[|k - k'|^2 + q_D^2]^2} = \frac{2\pi}{\hbar}\frac{N_I Z^2 q^4}{\Omega \epsilon_s^2}\frac{\delta(E_{k'} - E_k)}{[2k^2(1 - \cos\theta) + q_D^2]^2},$$

   where k and k' are measured relative to the bottom of the conduction band. N_I is the ionized impurity concentration, θ is the angle between k and k', and q_D is the inverse of the Debye length. Via integration, the rate of scattering through a given angle θ is

$$P(\theta, k) = \frac{\pi N_I Z^2 q^4 N(E_k)}{\hbar \epsilon_s^2}\frac{\sin\theta}{[2k^2(1 - \cos\theta) + q_D^2]^2},$$

   where $N(E_k)$ is the density of states within the band. It is possible to obtain the rate of momentum loss by integrating $P(\theta, k)(1 - \cos\theta)$ over possible values of θ (0 to π). The result is:

$$\frac{1}{\tau_m} = \frac{\pi N_I Z^2 q^4 N(E_k)}{\hbar \epsilon_s^2 k^4}\left[\ln\left|1 + \frac{4k^2}{q_D^2}\right| - \frac{4k^2/q_D^2}{1 + 4k^2/q_D^2}\right].$$

   (a) Plot the mobility as limited by ionized impurity scattering versus temperature for doping of 10^{16}cm^{-3} and 10^{18}cm^{-3} in silicon. Rather than integrating over all possible k values, you can instead use an average value of k for the given temperature ($KE = 3k_BT/2$). Assume the scattering is within a single minima which can be approximated to be spherically symmetric with a single effective mass $(m_l m_t^2)^{1/3}$. What is the approximate power law dependence?
   (b) Plot the mobility versus doping at room temperature. Comment on your results.