EE 531
Winter 2009

Homework #6 Solutions

1. (a) Threshold voltage of a symmetric, fully-depleted dual-gate MOSFET

By symmetry, $\mathcal{E}=0$ in center of thin Si region (at $x = x_s/2$).

Both channels are equivalent (turn on at same time) so focus on left ($x=0$)

net doping

from center of thin body $\rightarrow$ $\psi_s = (x_s/2)^2 \dfrac{qN_a}{2K_s\epsilon_0}$

$$V_{ox} = -\frac{Q_s/2}{C_{ox}'}$$ (half of Q_s' supported by each gate)

$$= +\frac{qN_a x_s/2}{C_{ox}'}$$

at EOSI, $\psi_I \approx 0.5V$

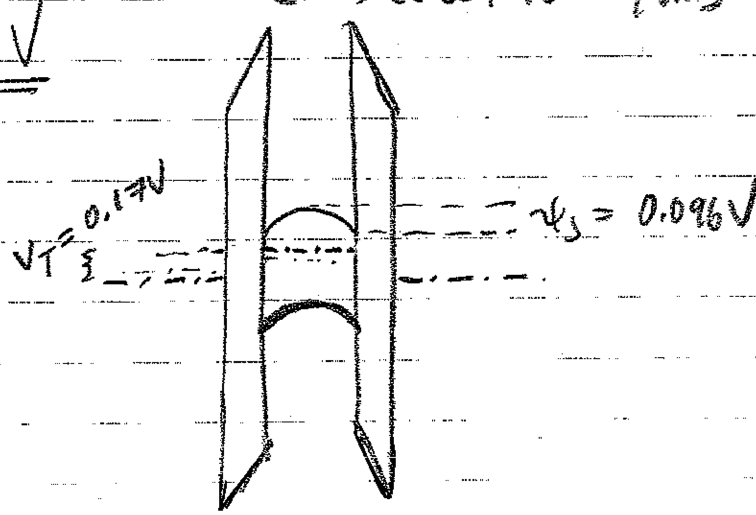
$$\Phi_{MS_I} = \Phi_M - \Phi_{S_I}$$
$$= \Phi_M - \left(X_s + \frac{E_g}{2q} - \psi_I\right)$$

$$V_T = \Phi_{MS_I} + V_{ox}$$

$$V_T = \Phi_M - 4.11V + \frac{qN_a x_s t_{ox}}{2K_{ox}\epsilon_0}$$

(b) $$V_T = 4.05 - 4.11 + \frac{q(2\times10^{19}\,cm^{-3})(5\times10^{-7}\,cm)(10^{-7}\,cm)}{2(3.9)(8.854\times10^{-14}\,F/cm)}$$
$$= \underline{\underline{0.17V}}$$

$\psi_s = 0.097$



2 (a) From Eq (18)

$$\cancel{\frac{d\langle p\rangle}{dt}} = -\frac{1}{m^*}\cancel{(\langle p\rangle\cdot\nabla_r)\langle \vec{p}\rangle} - q\mathcal{E} - \cancel{\frac{1}{n}\nabla_r(nkT_e)} - \frac{\langle \vec{p}\rangle}{\langle \tau_m\rangle}$$

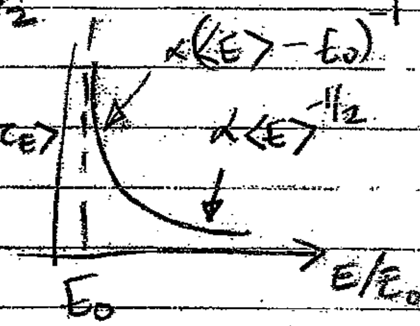
Steady-state Spatially uniform

$$\langle \vec{p}\rangle = -q\mathcal{E}\langle \tau_m\rangle = -q\mathcal{E}\left[A\langle E\rangle^{-3/2}\right]$$

$$|v_d| = \left|\frac{\langle p\rangle}{m^*}\right| = \frac{v_{dsat}\,\cancel{\mathcal{E}}}{\mathcal{E}+\mathcal{E}_c} = \frac{q\cancel{\mathcal{E}}A\langle E\rangle^{-3/2}}{m^*}$$

$$\langle E\rangle^{-3/2} = \frac{v_{dsat}\, m^*}{qA(\mathcal{E}+\mathcal{E}_c)}$$

$$\langle E\rangle = \left(\frac{qA(\mathcal{E}+\mathcal{E}_c)}{v_{dsat}\, m^*}\right)^{2/3}$$



(b) Use Eq. (19) to get $0 = -\frac{1}{m^*}q\mathcal{E}\cdot\langle p\rangle - \frac{\langle E\rangle - E_0}{\langle \tau_E\rangle}$

$$+\,q\mathcal{E}\left(\frac{q\mathcal{E}}{m^*}A\langle E\rangle^{-3/2}\right) = \frac{\langle E\rangle - E_0}{\langle \tau_E\rangle}$$

$$\langle \tau_E\rangle = \frac{(\langle E\rangle - E_0)\, m^*\langle E\rangle^{3/2}}{q^2\mathcal{E}^2 A} = \frac{(\langle E\rangle - E_0)\, m^*\langle E\rangle^{3/2}}{A\left(\frac{v_{dsat}\, m^*}{A}\langle E\rangle^{3/2} - q\mathcal{E}_c\right)^2}$$

From above,

$$\mathcal{E} = \frac{v_{dsat}\, m^*}{qA}\langle E\rangle^{3/2} - \mathcal{E}_c$$

For $\mathcal{E}=0$, $E = E_0$, so $E_0 = \left(\frac{qA\mathcal{E}_c}{v_{dsat}\, m^*}\right)^{2/3}$; $\mathcal{E}_c = \frac{v_{dsat}\, m^*}{qA}E_0^{3/2}$

$$\langle \tau_E\rangle = \frac{(\langle E\rangle - E_0)\, m^*\langle E\rangle^{3/2}}{\frac{(v_{dsat}\, m^*)^2}{A}\left(\langle E\rangle^{3/2} - E_0^{3/2}\right)^2}$$

See sketch above

$$\beta = \frac{4\left(3kT\, m^*\right) \epsilon_s kT}{\hbar^2 q^2 N_I} = \frac{12\left(kT/q\right)^2 m^* \epsilon_s}{\hbar^2 N_I}$$

$$= \frac{12\left(0.026\ \mathrm{V}\right)^2 \left(0.33\times 9.11\times 10^{-31}\ \mathrm{kg}\right)\left(8.854\times 10^{-14}\ \frac{\mathrm{F}}{\mathrm{cm}}\right) 12}{\left(1.05\times 10^{-34}\ \mathrm{J\,s}\right)^2 \left(10^{16}\ \mathrm{cm}^{-3}\right)} \qquad (1\ \mathrm{CV} = 1\ \mathrm{J})$$

$$= 2.4\times 10^{7}\ \frac{\mathrm{V}^2\,\mathrm{kg}\,\frac{\mathrm{C}}{\mathrm{V\,cm}}}{\mathrm{J}^2\mathrm{s}^2\,\mathrm{cm}^{-3}} = 2.4\times 10^{7}\ \frac{\cancel{\mathrm{kg}}}{\cancel{\mathrm{kg}}\,\mathrm{m}^2\,\mathrm{cm}^{-2}} = 2400$$

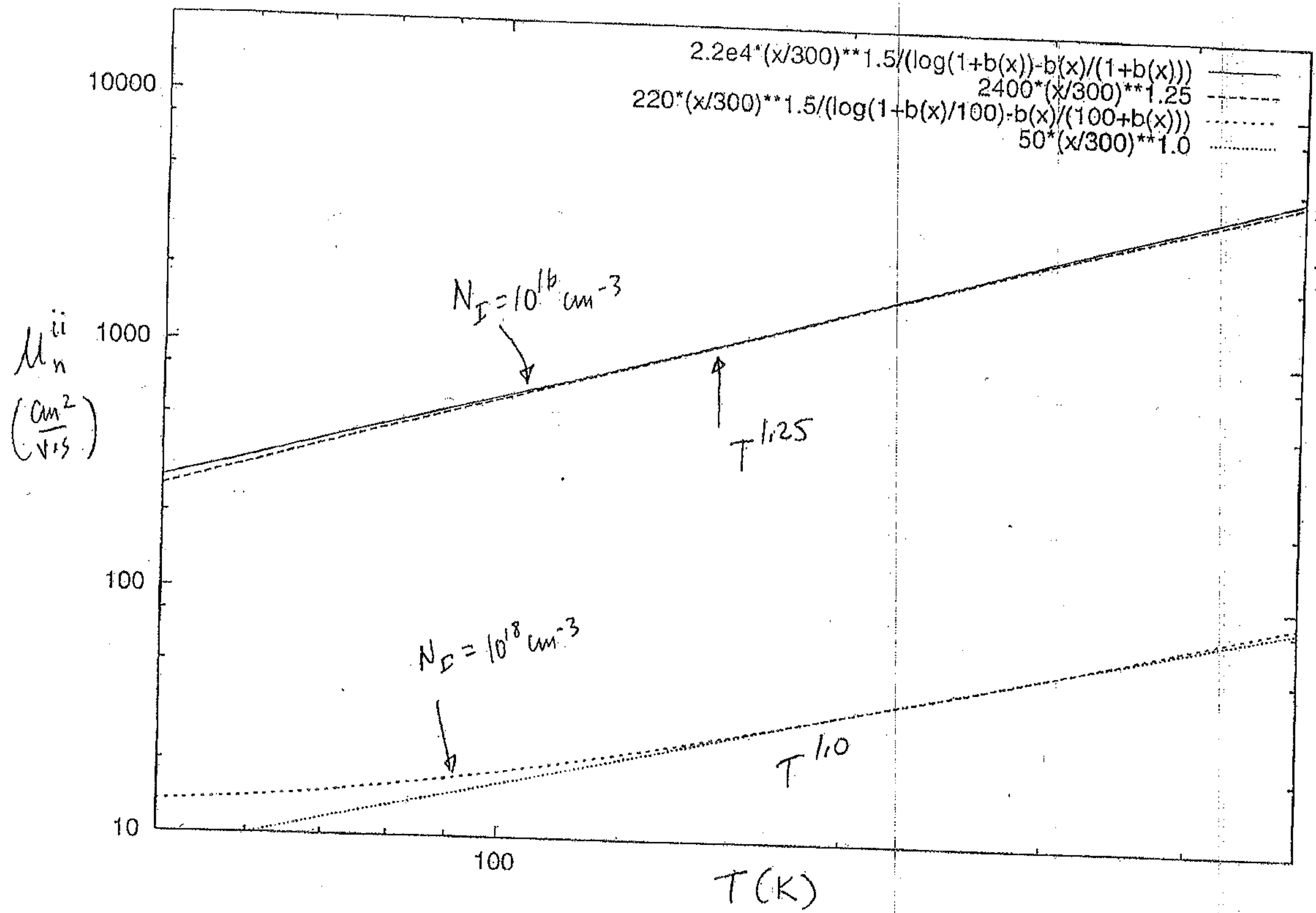
$$\beta = 2400\left[\frac{\left(T/300\mathrm{K}\right)^2}{\left(N_I/10^{16}\mathrm{cm}^{-3}\right)}\right], \qquad \mu = \frac{2.2\times 10^{4}\left(T/300\mathrm{K}\right)^{3/2}}{\left(N_I/10^{16}\mathrm{cm}^{-3}\right)\left[\ln|1+\beta| - \frac{\beta}{1+\beta}\right]}$$

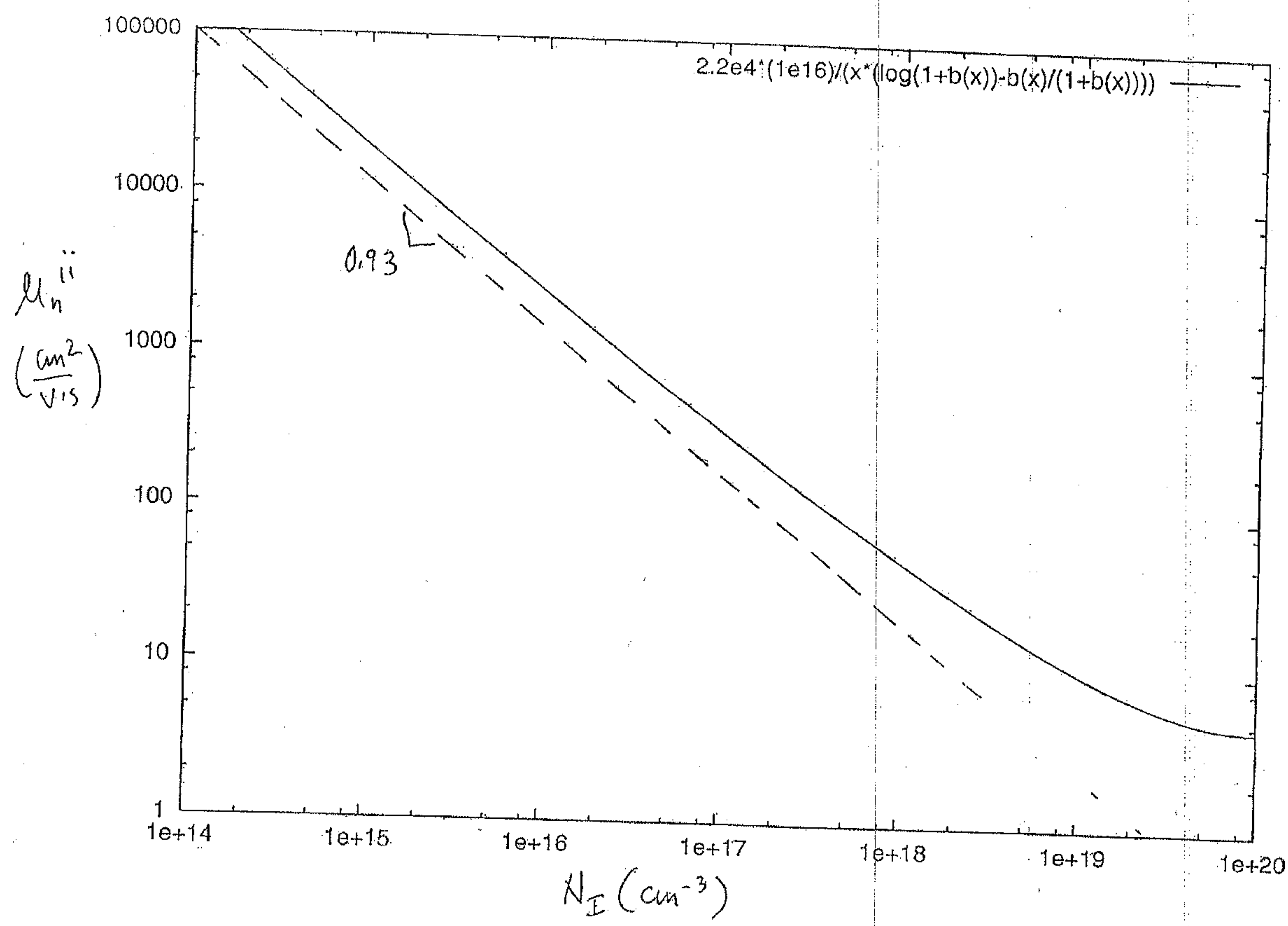
Looking at plot on next page; near 300K, $\mu \propto T^{1.25}$ for $N_I = 10^{16}$
$\mu \propto T$ for $N_I = 10^{18}$

As expected, mobility as limited by ionized impurity scattering increases with T due to higher speed of electrons. The power dependence varies with T and doping level. The dependence is slightly weaker than the simple $T^{3/2}$ dependence predicted by a simple model (leading term).

(b) Plot is shown on following page based on $\beta(T, N_I)$, $\mu(T, N_I)$ above. As expected, mobility drops with increased doping. The power dependence is slightly weaker than linear at low N_I, and saturates at high N_I.

Note that at low N_I or high T, μ will be limited by phonon rather than ionized impurity scattering.


2.2e4*(x/300)**1.5/(log(1+b(x))-b(x)/(1+b(x)))
2400*(x/300)**1.25
220*(x/300)**1.5/(log(1+b(x)/100)-b(x)/(100+b(x)))
50*(x/300)**1.0
10000
1000
100
10
100
μ_n^{ii} $\left(\frac{cm^2}{V \cdot s}\right)$
$N_I = 10^{16}\ cm^{-3}$
$T^{1.25}$
$N_I = 10^{18}\ cm^{-3}$
$T^{1.0}$
T (K)


2.2e4*(1e16)/(x*(log(1+b(x))-b(x)/(1+b(x))))
100000
10000
1000
100
10
1
1e+14
1e+15
1e+16
1e+17
1e+18
1e+19
1e+20
0.93
μ_n^{ii} $\left(\frac{cm^2}{V \cdot s}\right)$
N_I (cm^{-3})

3 $\mu = \frac{q\tau_m}{m^*}$

$$\frac{1}{\tau_m} = \frac{\pi N_I Z^2 q^4 N(E_k)}{\hbar \epsilon_s^2 k^4}\left[\ln\left|1+\frac{4k^2}{h_D^2}\right| - \frac{4k^2/h_D^2}{1+4k^2/h_D^2}\right]$$

$$N(E_k) = \frac{4\pi}{h^3}(2m^*)^{3/2}E_k^{1/2}, \text{ where } m^* = \overbrace{(m_l m_t^2)^{1/3}}^{=0.33m_0} \text{ and } E_k = E - E_c$$

$$h_D^2 = \frac{q^2 n_0}{\epsilon_s kT} \qquad k^2 = \frac{2E_k m^*}{\hbar^2} \qquad Z = 1 \text{ (singly ionized)}$$

a) Assume $n_0 = N_I$, and that on average $E_k = \frac{3}{2}kT$

Substituting above, $\mu = \frac{q\hbar\epsilon_s^2 k^4 h^3}{m^*\pi N_I Z^2 q^4 4\pi(2m^*)^{3/2}E_k^{1/2}\left[\ln|1+\beta| - \frac{\beta}{1+\beta}\right]}$

where $\beta = \frac{4k^2}{h_D^2}$ $= \frac{\hbar\epsilon_s^2\left(\frac{4E_k^2 m^{*2}}{\hbar^3}\right)h^3}{m^* N_I q^3(2\pi)^2(2m^*)^{3/2}E_k^{1/2}[\ \ ]}$

$$\beta = \frac{4(2E_k m^*)\epsilon_s(kT)}{\hbar^2 q^2 N_I}$$

$$\mu = \frac{\epsilon_s^2 4E_k^{3/2}(2\pi)}{m^{*1/2} N_I q^3 (2)^{3/2}[\ \ ]} \qquad (1J)^{1/2} = \left(\frac{kg\,m^2}{s^2}\right)^{1/2}$$

$$\mu = \frac{\epsilon_s^2 4(3kT)^{3/2}(2\pi)}{m^* N_I q^3 2^3[\ \ ]} = \frac{\epsilon_s^2 3^{3/2}(kT)^{3/2}\pi}{m^* N_I q^3[\ \ ]} \qquad \frac{C^{1/2}}{V^{1/2}}\frac{cm}{kg^{1/2}} = \frac{C^{1/2}V^{1/2}cm}{kg^{1/2}V}$$

note, we used $\frac{kT}{q}$

At $T = 300K$, $N_I = 10^{16}\,cm^{-3}$:

$$\mu = \frac{12^2(8.854\times10^{-14}F/cm)^2(3(0.026\,V))^{3/2}\pi}{(0.33\times9.11\times10^{-31}kg)^{1/2}(10^{16}cm^{-3})(1.6\times10^{-19}C)^{3/2}[\ \ ]} = \frac{220}{[\ \ ]}\;\frac{\frac{C^2}{V^2cm^2}V^{3/2}}{kg^{1/2}cm^{-3}C^{3/2}}$$

$$= 2.2\times10^4\,\frac{cm^2}{V\cdot s}$$