

Homework #7 Solutions

EE531
Spring 2009

$V_{gs} = 0$

$$1. \frac{I_{ds}^{sub}}{W} = \mu_{eff} C_{ox}' \frac{1}{L} (m-1) \left(\frac{kT}{q}\right)^2 \exp\left(-\frac{qV_T}{m kT}\right) < 10^{-9} \frac{A}{\mu m}$$

$$I_{ds}^{on} = I_{ds}^{sat} = \mu_{eff} C_{ox}' \frac{W}{L} \left(\frac{V_{gs} - V_T}{2m}\right)^2 \quad w/V_{gs} = 2V = V_{dd}$$

$$V_T = \phi_{ms} + 2\psi_B + \frac{qN_a W_{dm}}{C_{ox}'} + \frac{qN'}{C_{ox}'} - 24 \frac{t_{ox}}{W_{dm}} \sqrt{\psi_{bi}(2\psi_{bi} + V_{ds})} e^{-\frac{\pi L}{2m W_{dm}}}$$

$$W_{dm} = \sqrt{\frac{4K_s \epsilon_0 \psi_B}{qN_a}}, \quad m = 1 + \frac{3t_{ox}}{W_{dm}}$$

As rule of thumb: $W_{dm} > 6t_{ox} = 0.024 \mu m = 24 nm \Rightarrow N_a < 2.2 \times 10^{18} cm^{-3}$

$$L > 2(W_{dm} + 3t_{ox}) = 0.072 \mu m = 72 nm$$

Start with $N_a = \alpha * 10^{18} cm^{-3}$, $L = 0.1 \mu m + 0.001 \mu m * \beta = (100 + \beta) nm$

~~$W_{dm} =$~~ $\psi_B \approx 0.47 V$, $\phi_{ms} \approx -1.05 V$, $C_{ox}' = 8.6 \times 10^{-7} \frac{F}{cm^2}$

$$V_T = -0.11 V + 0.65 \alpha^{1/2} + 0.19 \gamma - 4.88 \alpha^{1/2} e^{-\pi(100+\beta)/2(\alpha^{1/2} + 0.34)35} \quad W_{dm} = 3.5 \times 10^{-6} cm (\alpha)^{-1/2} = 35 \alpha^{1/2} \mu m$$

$$= -0.11 + 0.65 \alpha^{1/2} + 0.19 \gamma - 0.17 \alpha^{1/2} e^{-3\beta/100}$$

$$N' = 10^{12} \gamma cm^{-2}$$

$$m = 1 + 0.34 \alpha^{1/2} \approx 1.34$$

$$= \pi(100+\beta)/70(1.34) \quad \alpha \approx 1 + \epsilon$$

$$e^{-\frac{\pi(100+\beta)}{70(1.34)}} = 0.035 * e^{-3\beta}$$

$$\epsilon_{eff} \approx \frac{1Qd}{K_s \epsilon_0}, \quad \epsilon_{eff}^{on} \approx \frac{1Qd + \frac{1}{2} C_{ox}' (V_{gs} - V_T)}{K_s \epsilon_0} = 0.54 \alpha^{1/2} \frac{MV}{cm} + 0.66 \frac{MV}{cm}$$

$$\approx \frac{q W_{dm} N_a}{K_s \epsilon_0} = 5.4 \alpha^{1/2} \times 10^{15} \frac{V}{cm} \Rightarrow \mu_{eff} = \frac{100 cm^2}{V \cdot s}$$

$$\mu_{eff}^{on} \approx 180 \frac{cm^2}{V \cdot s}$$

From Fig 3.13

For off current, worst case is $N_a^{wc} = N_a(0.95)$

$$N_{wc}' = N'(0.95)$$

$$L^{wc} = L - 20 \text{ nm}$$

$$\frac{I_{ds}^{sub}}{W} = 100 \frac{\text{cm}^2}{\text{V}^2} \left(8.6 \times 10^{-7} \frac{\text{F}}{\text{cm}^2} \right) \frac{1}{(0.1 + \beta/1000)} \cdot (0.34 \alpha^{1/2}) \left(\frac{kT}{q} \right)^2$$

* $\exp\left(\frac{-qV_T}{(1+0.34\alpha^{1/2})kT}\right)$

Assume $\alpha = 0.95$ (worst case effect) and $\beta = -20$

$$V_T^{wc} \approx 0.19 \text{ V}$$

$$V_T^{nom} (\alpha=1, \beta=0) = 0.37 + 0.198$$

$$V_T^{wc} (\alpha=0.95, \beta=-20) = 0.22 + 0.198$$

For $\gamma = 0$, could increase $|\beta|$ to 22 and have $V_T^{wc} = 0.19 \text{ V}$

This corresponds to $L_{nom} = 97 \text{ nm}$

For on current, worst-case is $N_a^{wc} = 1.05 N_a$

$$N_{wc}' = 1.05 N'$$

$$L^{wc} = L + 20 \text{ nm}$$

$$\frac{I_{on}}{W} = \frac{I_{ds}^{sat}}{W} = \mu_{eff} C_{ox}' \frac{1}{L} \frac{(V_{gs} - V_T^{wc})^2}{2m} = 180 \frac{\text{cm}^2}{\text{V}\cdot\text{s}} (8.6 \times 10^{-7} \frac{\text{F}}{\text{cm}^2}) \frac{1}{0.12 \mu\text{m}} \frac{(2 - 0.46)^2}{2(1.34)}$$

$$= 11.4 \text{ A/cm} = 1.14 \times 10^{-3} \text{ A}/\mu\text{m}$$

$$V_T^{wc} = -0.11 + 0.65(1.05)^{1/2} - 0.17(1.05)^{1/2} e^{-60/100} = 0.46 \text{ V}$$

$$\tau = \frac{(4L C_{ox}' + 0.5 \mu\text{m} C') 2V}{I_{ds}^{on} / W} \quad C' = 4 \times 10^{-7} \frac{\text{F}}{\text{cm}^2}$$

$$= \frac{[4(0.12 \times 10^{-4} \text{ cm}) (8.6 \times 10^{-7} \frac{\text{F}}{\text{cm}^2}) + 5 \times 10^{-5} \text{ cm} (4 \times 10^{-7} \frac{\text{F}}{\text{cm}^2})] 2V}{11.4 \text{ A/cm}} = 1.1 \times 10^{-14} \text{ s}$$

$$= 11 \text{ fs}$$

$$W / v_{sat} \frac{\mu_{eff} V_{ds}}{v_{sat} L} = \frac{(180 \frac{\text{cm}^2}{\text{V}\cdot\text{s}})(2V)}{(8 \times 10^6 \frac{\text{cm}}{\text{s}})(0.12 \mu\text{m})} = 3.75 \text{ so}$$

Using 3.78 instead;

velocity saturation is significant

$$\frac{I_{dsat}}{W} = C_{ox}' v_{sat} (V_{gs} - V_T) \frac{\sqrt{1 + \xi} - 1}{\sqrt{1 + \xi} + 1} \quad \xi = 2.15$$

$$= 2.96 \text{ A/cm}, \quad \tau = 4.2 \times 10^{-14} \text{ s} = 42 \text{ fs}$$

To optimize, check whether lower ^(or higher) doping would give better results (e.g., try $8 \times 10^{17} \text{ cm}^{-3}$ or $1.2 \times 10^{18} \text{ cm}^{-3}$) and repeat.

2 T & N 4.10 $Q_i(V) = -q \int_0^{\psi_s} \frac{(n_i^2/N_a) e^{\psi(\psi-V)/kT}}{\epsilon(\psi, V)} d\psi$

In subthreshold regime, $\epsilon \approx$ constant across inversion layer, so

$$Q_i(V) \approx -\frac{q}{\epsilon_s(\psi_s, V)} \left(\frac{n_i^2}{N_a}\right) \int_0^{\psi_s} e^{\psi(\psi-V)/kT} d\psi$$

$$\approx -\frac{q}{\epsilon_s(\psi_s, V)} \frac{n_i^2}{N_a} \frac{kT}{q} e^{\psi(\psi-V)/kT} \Big|_0^{\psi_s}$$

$$\approx -\frac{kT n_i^2}{\epsilon_s N_a} e^{\psi_s^2/kT} \quad \text{for } V \text{ small}$$

$$I_{ds} = \mu_{eff} W \frac{dV}{dy} Q_i \Rightarrow \frac{1}{I_{ds}} dV = \frac{1}{\mu_{eff} W Q_i} dy$$

$$\frac{V_{ds}}{I_{ds}} = \frac{1}{\mu_{eff} W} \int_0^L \frac{1}{Q_i(y)} dy$$

$$= \frac{N_a}{\mu_{eff} W kT n_i^2} \int_0^L \underbrace{\epsilon_s(y)}_{\text{dominated by region w/ smallest } \psi_s} e^{-\psi_s^2/kT} dy$$

Integral dominated by region w/ smallest ψ_s

Effective channel length is length of region w/ smallest ψ_s .

If $Q_i(y)$ want to integrate Q_i over y rather than V .

3 T & N S. 4 $\tau_{tr} = \left| \frac{Q}{I} \right|$ or $|I| = \frac{|Q|}{\tau_{tr}}$ for any transport problem

$$\tau_{tr} = \frac{-\cancel{W}L\cancel{C_{ox}}(V_g - V_t) \sqrt{\frac{+1/3}{+1}}}{\cancel{C_{ox}}\cancel{W}v_{sat}(V_g - V_t) \sqrt{\frac{-1}{+1}}}$$

$$\tau_{tr} = \frac{L}{v_{sat}} \frac{\sqrt{+1/3}}{\sqrt{-1}} \quad \text{where } \sqrt{\quad} = \sqrt{1 + \frac{2\mu_{eff}(V_g - V_t)}{m v_{sat} L}}$$

$$\text{In } L \rightarrow 0 \quad \sqrt{\quad} \rightarrow \sqrt{\frac{2\mu_{eff}(V_g - V_t)}{m v_{sat} L}} \gg 1$$

$\tau_{tr} \rightarrow \frac{L}{v_{sat}}$ as expected since entire channel is at saturation velocity