EE-612:
Lecture 6
MOSFET IV: Part II

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NCN

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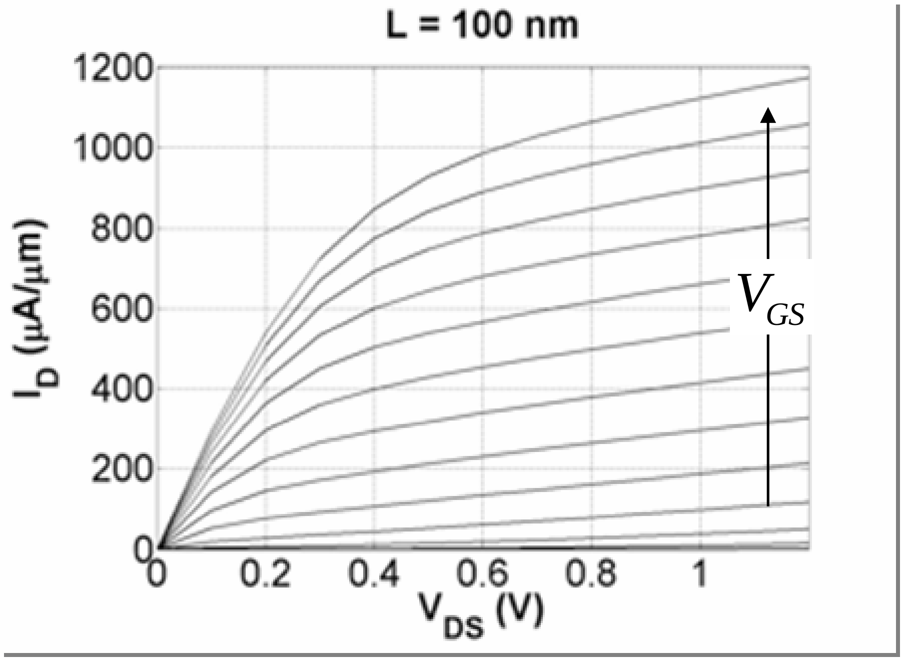
outline

1) **Review**
2) Bulk charge theory (approximate)
3) Velocity saturation theory
4) Summary

typical Si NMOS characteristics



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$I_D \propto W\left(V_{GS} - V_T\right)^1$$

MOSFET IV: summary

linear region: $V_{DS} << V_{DSAT}$

$$I_D = \frac{W}{L}\mu_{eff}C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

saturated region: $V_{DS} > V_{DSAT}$

$$I_D = \frac{W}{2L}\mu_{eff}C_{ox}\left(V_{GS} - V_T\right)^2 \qquad V_{DS} = V_{GS} - V_T$$

$$I_D = W\,C_{ox}\,\upsilon_{sat}\left(V_{GS} - V_T\right) \qquad V_{DS} > V_{SAT}$$

See: “A Review of MOSFET Fundamentals,” M. Lundstrom,
http://nanohub.edu/resources/5307

square law theory

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_I(V)\,dV$$

$$Q_I(y) = -C_{ox}\left[V_G - V_T(y)\right]$$

$$Q_I(y) = -C_{ox}\left[V_G - V_T - V(y)\right]$$

$$V_{GS} > V_T \quad V_{DS} \le V_{GS} - V_T$$

$$I_D = \mu_{eff} C_{ox}\frac{W}{L}\left[\left(V_{GS} - V_T\right)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$V_{GS} > V_T \quad V_{DS} > V_{GS} - V_T$$

$$I_D = \mu_{eff} C_{ox}\frac{W}{2L'}\left(V_{GS} - V_T\right)^2$$

bulk charge theory

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_I(V)dV$$

$$Q_I(y) = -C_{ox}\left[V_G - V_T(y)\right]$$

$$V_T(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\varepsilon_{Si}N_A\left(2\psi_B + V(y)\right)}\Big/C_{ox}$$

bulk charge theory

$$V_{GS} > V_T \quad V_{DS} \leq V_{GS} - V_T$$

$$I_D = \mu_{eff} C_{ox} \frac{W}{L} \left[\left(V_{GS} - V_{FB} - 2\psi_B - \frac{V_{DS}}{2} \right) V_{DS} - \frac{2\sqrt{2\varepsilon_{Si} q N_A}}{3C_{ox}} \left[\left(2\psi_B + V_{DS}\right)^{3/2} - \left(2\psi_B\right)^{3/2} \right] \right]$$

eqn. (3.18) Taur and Ning

expand for small V_{DS}:

$$I_D = \mu_{eff} C_{ox} \frac{W}{L} \left(V_{GS} - V_T \right) V_{DS}$$

bulk charge theory: ii

$$V_{GS} > V_T \qquad V_{DS} \leq \left(V_{GS} - V_T\right)/m$$

$$I_D = \mu_{eff} C_{ox} \frac{W}{L} \left[\left(V_{GS} - V_{FB} - 2\psi_B - \frac{V_{DS}}{2} \right) V_{DS} - \frac{2\sqrt{2\varepsilon_{Si} q N_A}}{3C_{ox}} \left[\left(2\psi_B + V_{DS}\right)^{3/2} - \left(2\psi_B\right)^{3/2} \right] \right]$$

expand for larger V_{DS} :

$$I_D = \mu_{eff} C_{ox} \frac{W}{L} \left[\left(V_{GS} - V_T\right) V_{DS} - \frac{m}{2} V_{DS}^2 \right] \qquad m = 1 + \frac{\sqrt{\varepsilon_{Si} q N_A / 4\psi_B}}{C_{ox}}$$

bulk charge theory: iii

$$V_{GS} > V_T \quad V_{DS} > \left(V_{GS} - V_T\right)/m$$

$$I_D = \mu_{eff} C_{ox} \frac{W}{L}\left[\left(V_{GS} - V_T\right)V_{DS} - \frac{m}{2}V_{DS}^2\right] \rightarrow$$

$$I_D = \mu_{eff} C_{ox} \frac{W}{L'} \frac{\left(V_{GS} - V_T\right)^2}{2m}$$

bulk charge theory: summary

$$V_{GS} > V_T \quad V_{DS} \le (V_{GS} - V_T)/m$$

$$I_D = \mu_{eff} C_{ox} \frac{W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{m}{2} V_{DS}^2 \right] \qquad m = 1 + \frac{\sqrt{\varepsilon_{Si} q N_A / 4\psi_B}}{C_{ox}}$$

$$V_{GS} > V_T \quad V_{DS} > (V_{GS} - V_T)/m$$

$$I_D = \mu_{eff} C_{ox} \frac{W}{L'} \frac{(V_{GS} - V_T)^2}{2m}$$

How can we derive these results more simply and give a physical interpretation to *m*?

outline

1) Review
2) **Bulk charge theory (approximate)**
3) Velocity saturation theory
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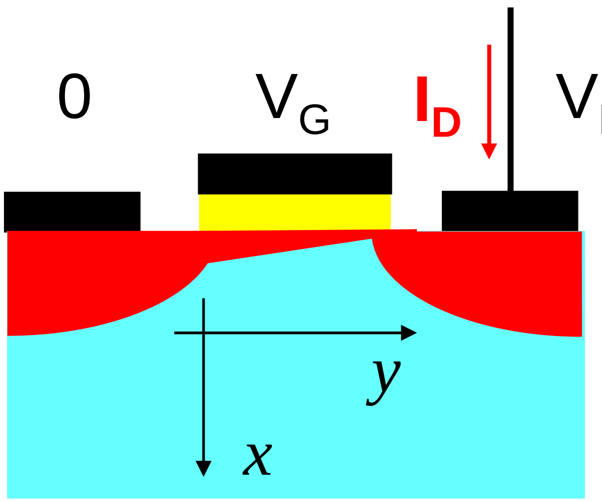
I-V formulation



$$I_D = W\,Q_I(y)\,\upsilon_y(y)$$

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_I(V)\,dV$$

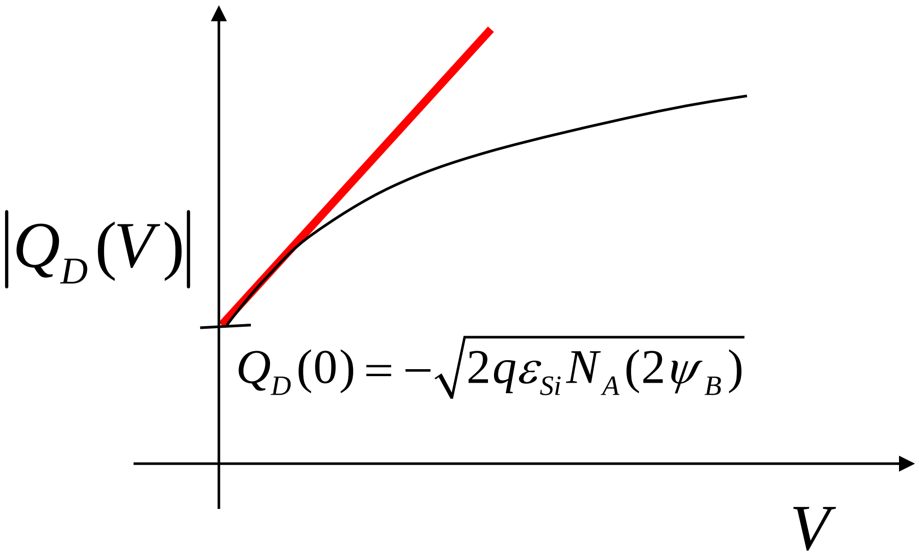
$$Q_I(y) = -C_{ox}\left[V_G - V_T(y)\right]$$

$$V_T(y) = V_{FB} + 2\psi_B + V(y) - Q_D(V)/C_{ox}$$

$$V_T(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\varepsilon_{Si}N_A\left(2\psi_B + V(y)\right)}/C_{ox}$$

approximate Q_D

$$Q_D(V) = -\sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V)}$$



can we use a linear approximation for Q_D?

approximate Q_D

$$Q_D(V) = -\sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V)}$$

$$Q_D(V) = Q_D(0) + \left.\frac{dQ_D}{dV}\right|_{V=0} V + \ldots$$

$$\left.\frac{dQ_D}{dV}\right|_{V=0} = -\frac{\varepsilon_{Si}}{W_{DM}} = -C_{DM}$$

$$Q_D(V) = Q_D(2\psi_B) - C_{DM}V$$

approximate Q_I

$$Q_I(y) = -C_{ox}\left[V_G - V_T(y)\right] \qquad V_T(y) = V_{FB} + 2\psi_B + V(y) - Q_D\left(V\right)/C_{ox}$$

$$Q_D(V) = Q_D(2\psi_B) - C_{DM}V$$

$$Q_I(V) = -C_{ox}\left(V_G - V_{FB} - 2\psi_B + \frac{Q_D(2\psi_B)}{C_{ox}} - V - \frac{C_{DM}}{C_{ox}}V\right)$$

$$\underbrace{V_{FB} - 2\psi_B + \frac{Q_D(2\psi_B)}{C_{ox}}}_{-V_T} \qquad \underbrace{V - \frac{C_{DM}}{C_{ox}}V}_{-\left(1 + C_{DM}/C_{ox}\right)V}$$

$$Q_I(y) = -C_{ox}\left(V_G - V_T - mV\right)$$

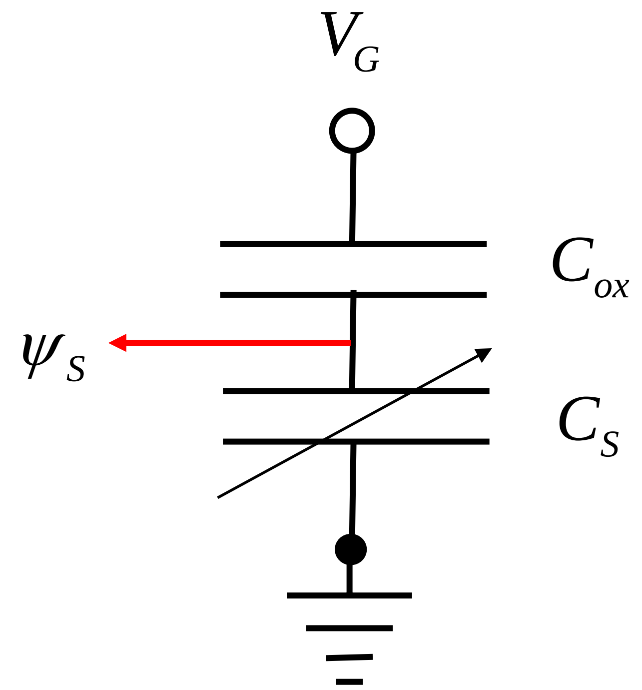
$$m = \left(1 + C_{DM}/C_{ox}\right)$$

meaning of *m*

$$m = \left(1 + C_{DM}/C_{ox}\right)$$

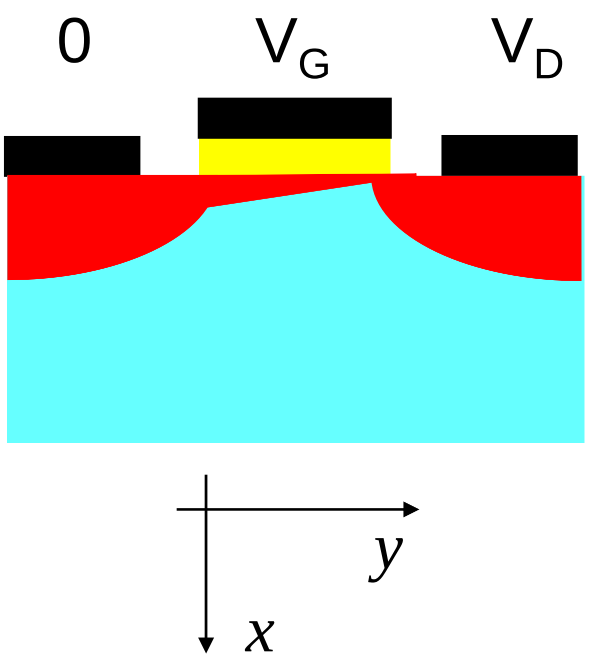
'body effect coefficient'

$$m = \left(1 + 3t_{ox}/W_{DM}\right)$$



$$\Delta\psi_S = \frac{C_{ox}}{C_{ox} + C_{DM}}\Delta V_G = \frac{\Delta V_G}{m}$$

IV relation

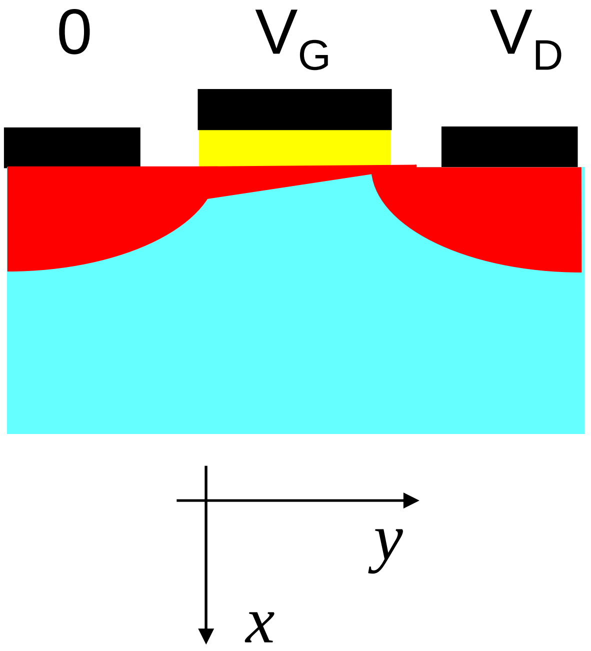


$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_I[V]\,dV$$

$$I_D = \mu_{eff}C_{ox}\frac{W}{L}\int_0^{V_D}\left[V_G - V_T - mV\right]dV$$

$$I_D = \mu_{eff}C_{ox}\frac{W}{L}\left[\left(V_{GS} - V_T\right)V_{DS} - \frac{m}{2}V_{DS}^2\right]$$

pinch-off



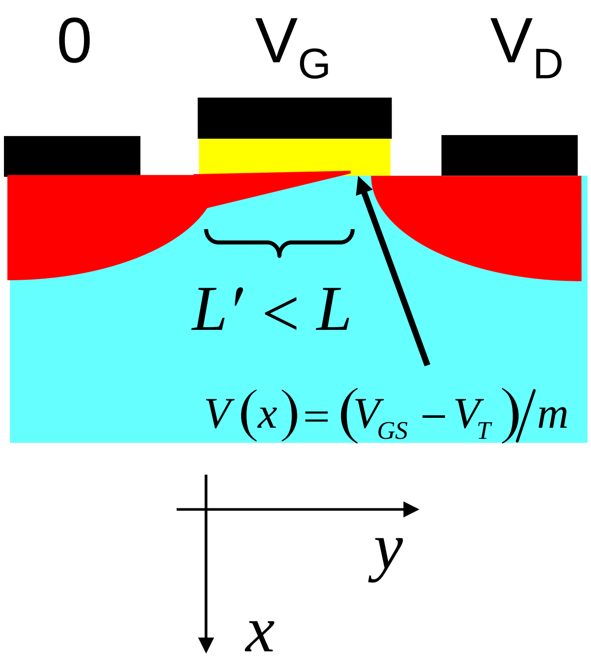
$$Q_I(L) = -C_{ox}\left[V_G - V_T - mV_D\right]$$

when $V_D = \left(V_G - V_T\right)/m,$

then $Q_I(L) = 0$

$E_y >> E_x$ GCA fails!

beyond pinch-off, $V_{DS} > V_{DSAT}$



channel is pinched-off near the drain but current still flows.

$$I_D \approx I_D\left(V_{DS} = \left(V_{GS} - V_T\right)/m\right)$$

$$I_D = \mu_{eff} C_{ox} \frac{W}{2L'} \frac{\left(V_{GS} - V_T\right)^2}{m}$$

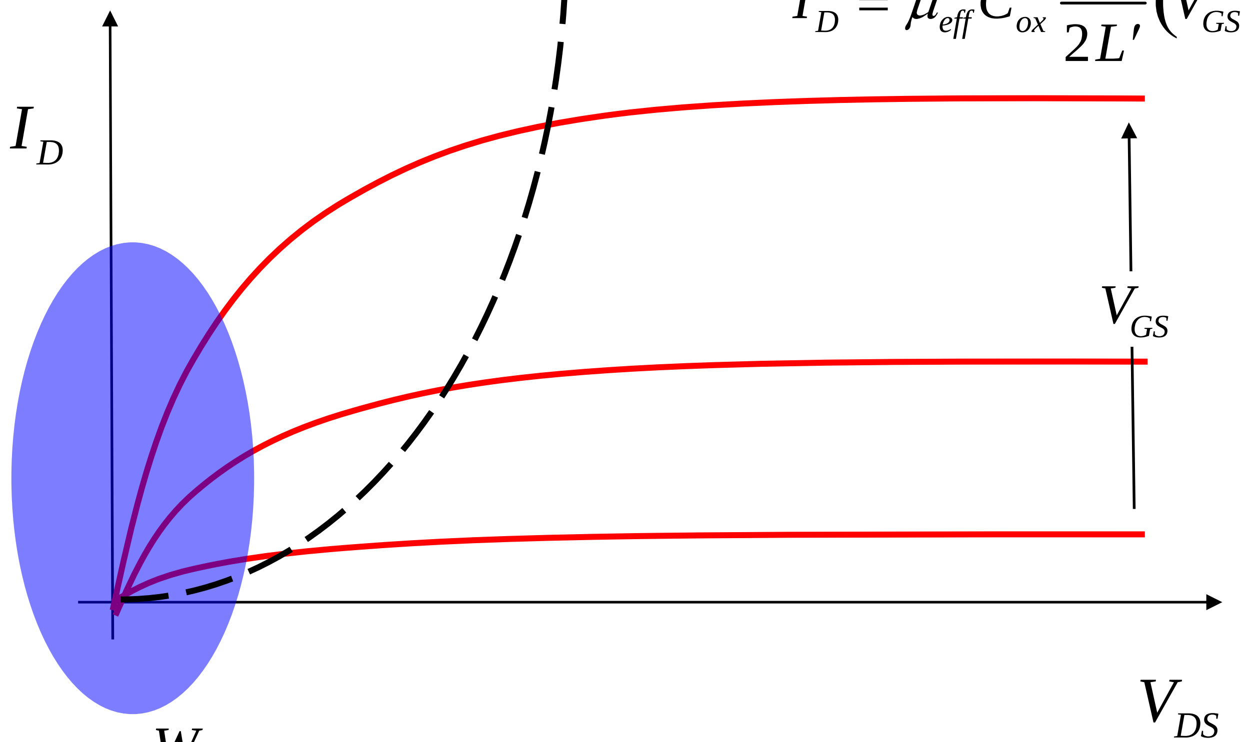
$$V_{GS} > V_T$$

$$V_{DS} > \left(V_{GS} - V_T\right)/m$$

IV summary

$$V_{DSAT} = \left(V_{GS} - V_T\right) / m$$

$$I_D = \mu_{eff} C_{ox} \frac{W}{2L'} \left(V_{GS} - V_T\right)^2 / m$$



$$I_D = \mu_{eff} C_{ox} \frac{W}{L'} \left(V_{GS} - V_T\right) V_{DS}$$

outline

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3) Velocity saturation theory

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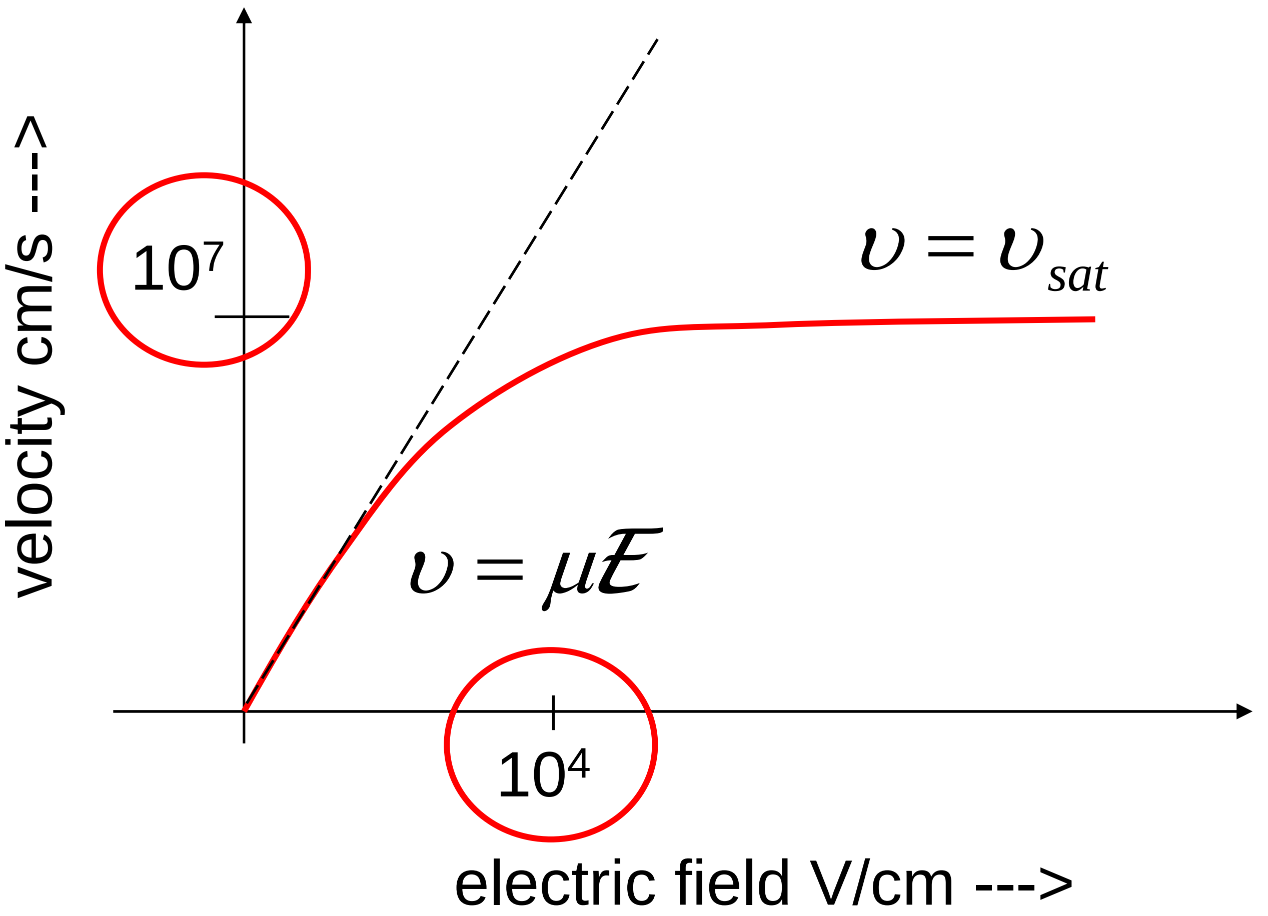
velocity saturation in bulk silicon



velocity saturation and MOSFETs

$$I_D = W\,Q_I(y)\upsilon_y(y)$$

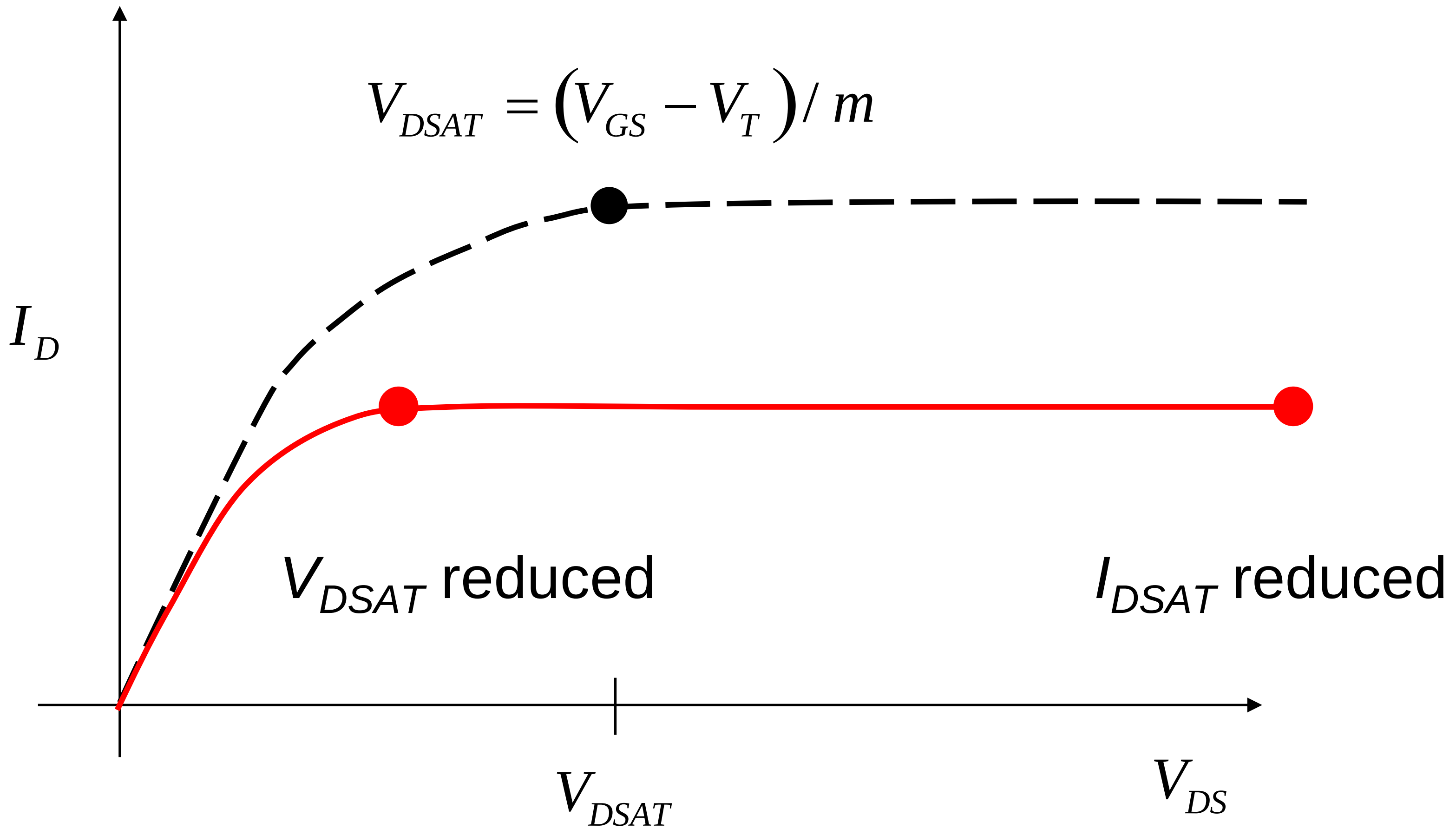
$$\upsilon_y(y) = \mu_{eff}E_y(y)\,?$$

$$E_y \sim \frac{V_{DD}}{L} << 10^4 \text{ V/cm}$$

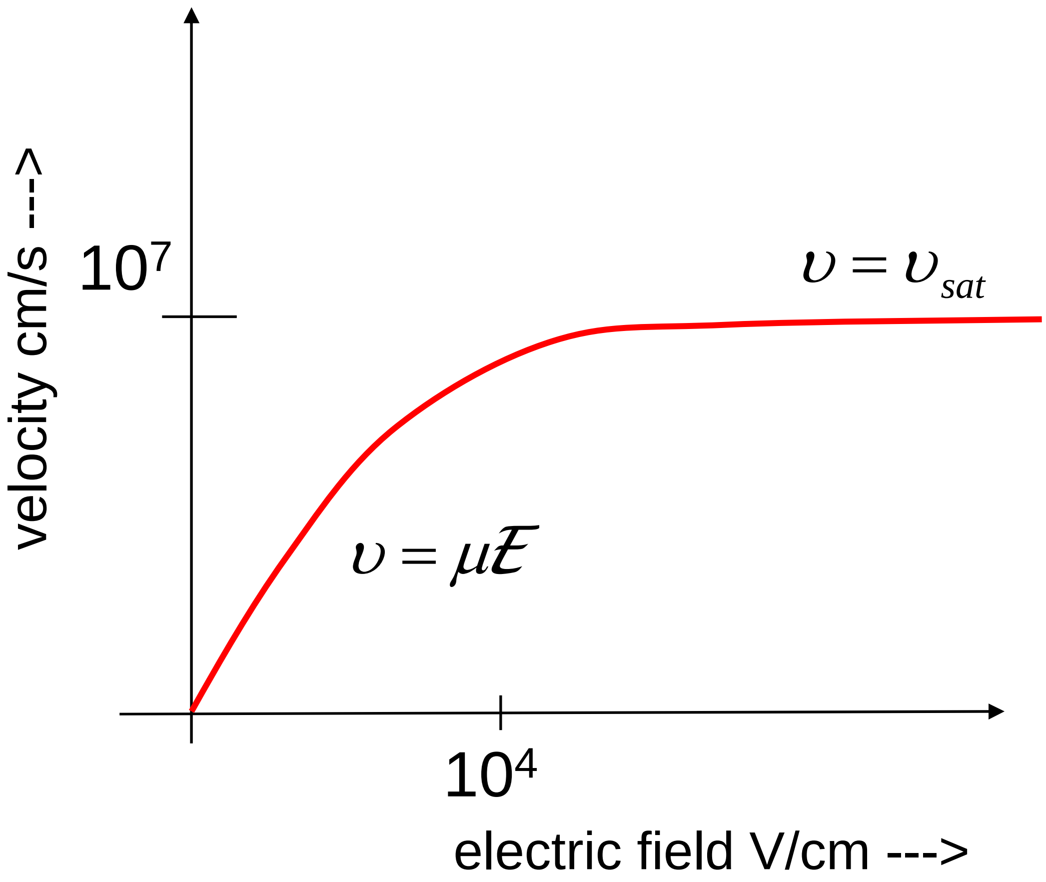
OK for *L* >> 1 micrometer

$$L >> \frac{V_{DD}}{10^4}$$

expected result



velocity vs. field characteristic (electrons)

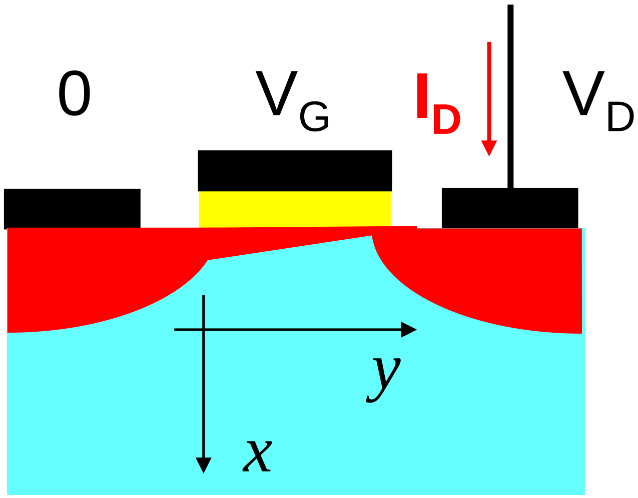


$$\upsilon_d = \frac{-\mu E}{\left[1+(E/E_c)^2\right]^{1/2}}$$

$$\upsilon_d = \frac{-\mu E}{1+(|E|/E_c)}$$

$$\mu E_C = \upsilon_{sat}$$

I-V derivation



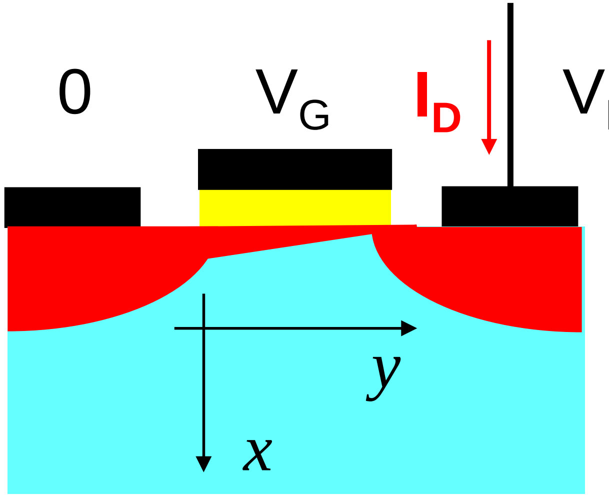
$$I_D = -W\,Q_I\left(y\right)\upsilon_y(y)$$

$$\upsilon(y) = \frac{-\mu_{eff} E}{1 + (|E|/E_c)}$$

$$I_D = WQ_I\mu_{eff}\,\frac{E_y}{1 + |E_y|/E_c}$$

$$I_D\left(1 + \frac{1}{E_c}\frac{dV}{dy}\right) = -WQ_I\mu_{eff}\,\frac{dV}{dy}$$

I-V derivation: ii



$$I_D\left(1+\frac{1}{E_c}\frac{dV}{dy}\right)=-WQ_I\mu_{eff}\frac{dV}{dy}$$

$$I_D\left(1+\frac{1}{E_c}\frac{dV}{dy}\right)dy=-WQ_I\mu_{eff}dV$$

$$I_D\left\{\int_0^L dy+\int_0^{V_{DS}}+\frac{1}{E_c}dV\right\}=-\int_0^{V_{DS}}WQ_I\mu_{eff}dV$$

$$I_D L\left\{1+V_{DS}/LE_c\right\}$$

same as before

derivation (iii)

$$I_D = F_v \mu_{eff} C_{ox} \frac{W}{L}\left[\left(V_{GS} - V_T\right)V_{DS} - m\frac{V_{DS}^2}{2}\right] \qquad (1)$$

$$F_v = \frac{1}{\left(1 + V_{DS}/LE_c\right)} = \frac{1}{\left(1 + \mu_{eff} V_{DS}/\upsilon_{sat} L\right)}$$

$V_{DS}/L =$ average electric field in the channel

when $V_{DS}/L >> E_c$ then $F << 1$

(1) *valid when*:

$V_{GS} > V_T \quad V_{DS} < ?$

V_{DSAT}

$$\frac{dI_D}{dV_{DS}} = 0$$

$$V_{DSAT} = \frac{2\left(V_{GS} - V_T\right)/m}{1 + \sqrt{1 + 2\mu_{eff}\left(V_{GS} - V_T\right)/m\upsilon_{sat}L}} < \frac{\left(V_{GS} - V_T\right)}{m}$$

eqn. (3.77) of Taur and Ning

I_{DSAT}

$$I_{DSAT} = W\,C_{ox}\upsilon_{sat}\left(V_{GS}-V_T\right)\frac{\sqrt{1+2\mu_{eff}\left(V_{GS}-V_T\right)/m\upsilon_{sat}L}-1}{\sqrt{1+2\mu_{eff}\left(V_{GS}-V_T\right)/m\upsilon_{sat}L}+1}$$

eqn. (3.78) of Taur and Ning

Examine two limits:

i) $L \to \infty$

ii) $L \to 0$

L --> inf

$$V_{DSAT} \rightarrow \frac{\left(V_{GS} - V_T\right)}{m}$$

$$I_{DSAT} \rightarrow \mu_{eff} C_{ox} \frac{W}{2L} \frac{\left(V_{GS} - V_T\right)^2}{m}$$

L --> 0

$$V_{DSAT} \rightarrow \sqrt{2\upsilon_{sat} L \left(V_{GS} - V_T\right) / m\mu_{eff}}$$

$$I_{DSAT} = W\, C_{ox} \upsilon_{sat} \left(V_{GS} - V_T\right)$$

"complete velocity saturation"

current independent of *L*

near threshold

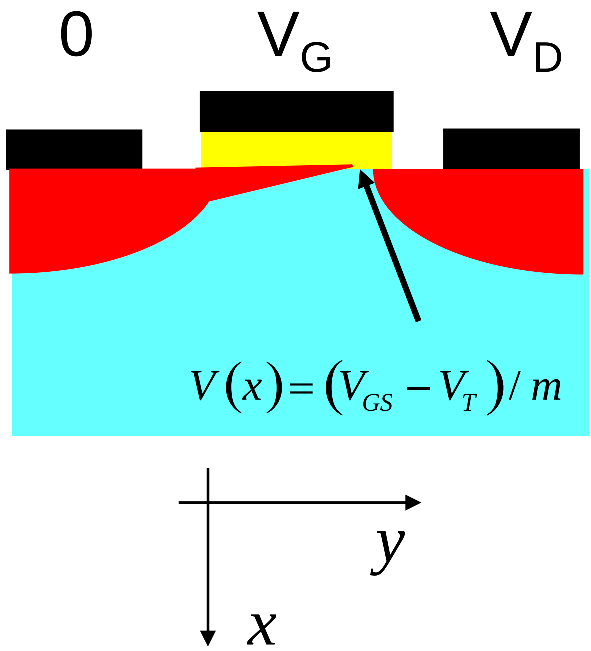
$$\frac{2\mu_{eff}\left(V_{GS}-V_T\right)}{m\upsilon_{sat}L} << 1$$

$$V_{DSAT} \rightarrow \left(V_{GS}-V_T\right)/m$$

$$I_{DSAT} \rightarrow \mu_{eff} C_G \frac{W}{2L} \frac{\left(V_{GS}-V_T\right)^2}{m}$$

near threshold is like long channel

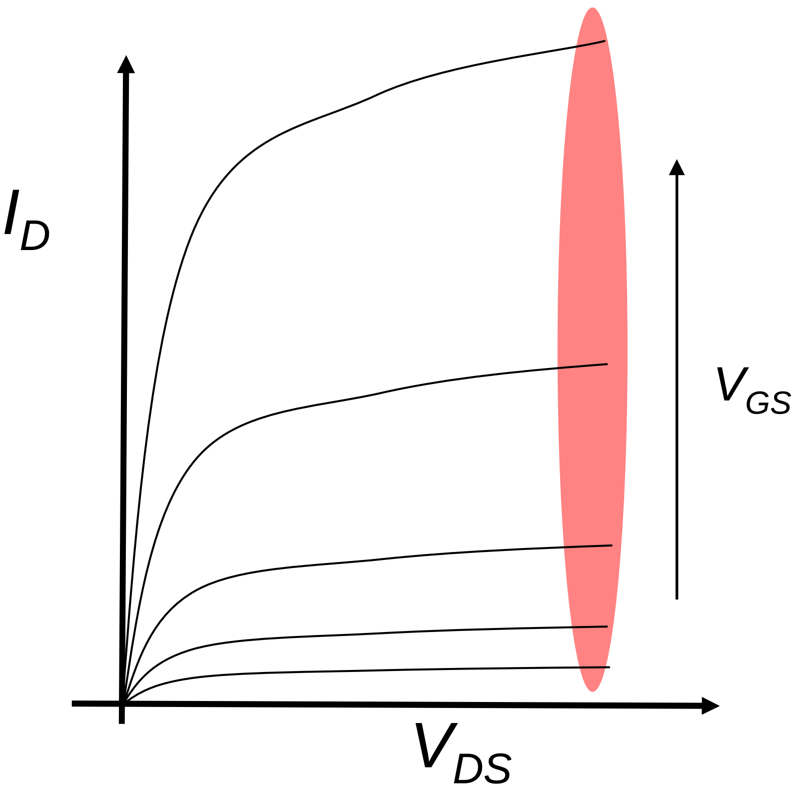
near threshold

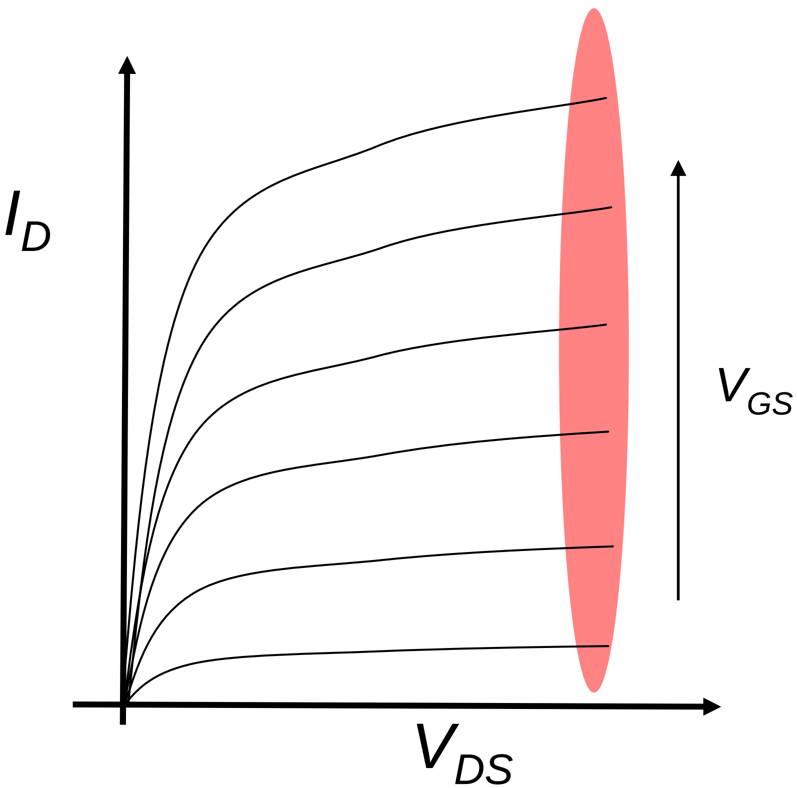


$$\frac{2\mu_{eff}(V_{GS} - V_T)}{m\upsilon_{sat} L} << 1$$

$$\frac{(V_{GS} - V_T)/m}{L} < \frac{E_c}{2}$$

‘signature’ of velocity saturation

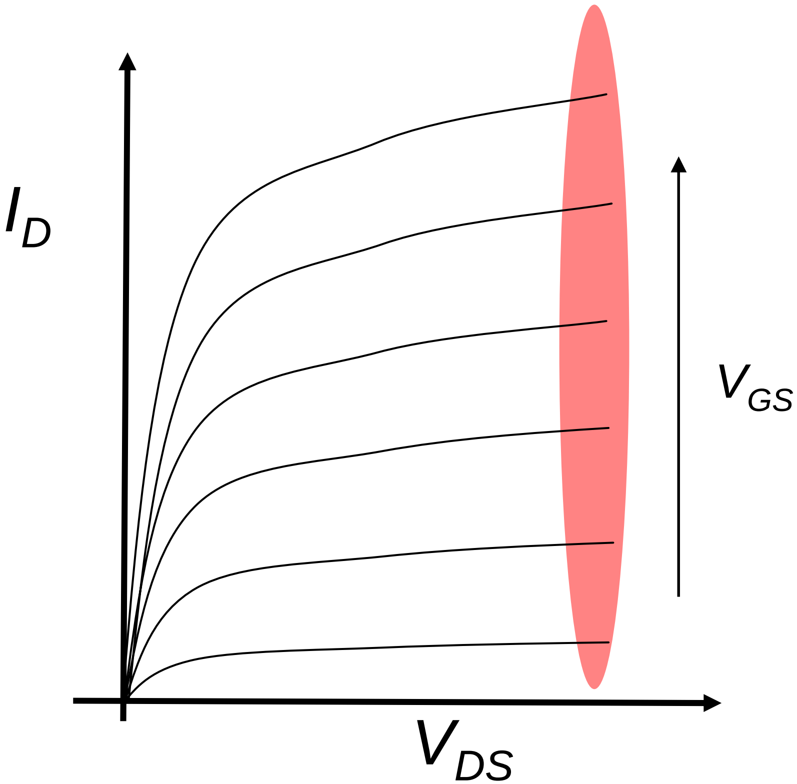




$$I_D = \frac{W}{2L}\mu_{eff}C_{ox}\frac{\left(V_{GS}-V_T\right)^2}{m}$$

$$I_D = W\upsilon_{sat}C_{ox}\left(V_{GS}-V_T\right)$$

I_D and $(V_{GS} - V_T)$



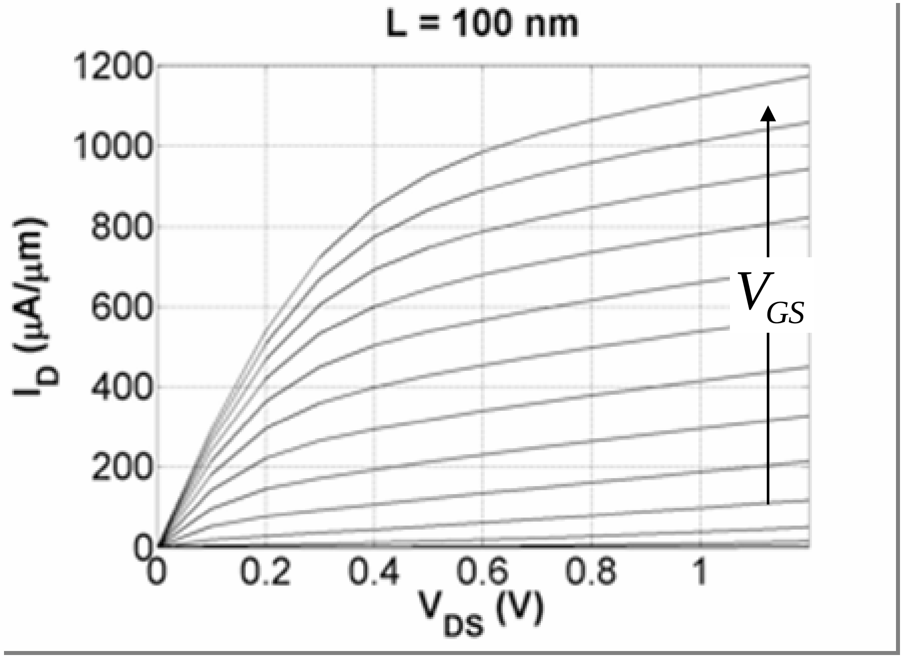
$$I_D(V_{DS} = V_{DD}) \sim \left(V_{GS} - V_T\right)^{\alpha}$$

$$1 < \alpha < 2$$

complete velocity saturation

long channel

typical Si NMOS characteristics

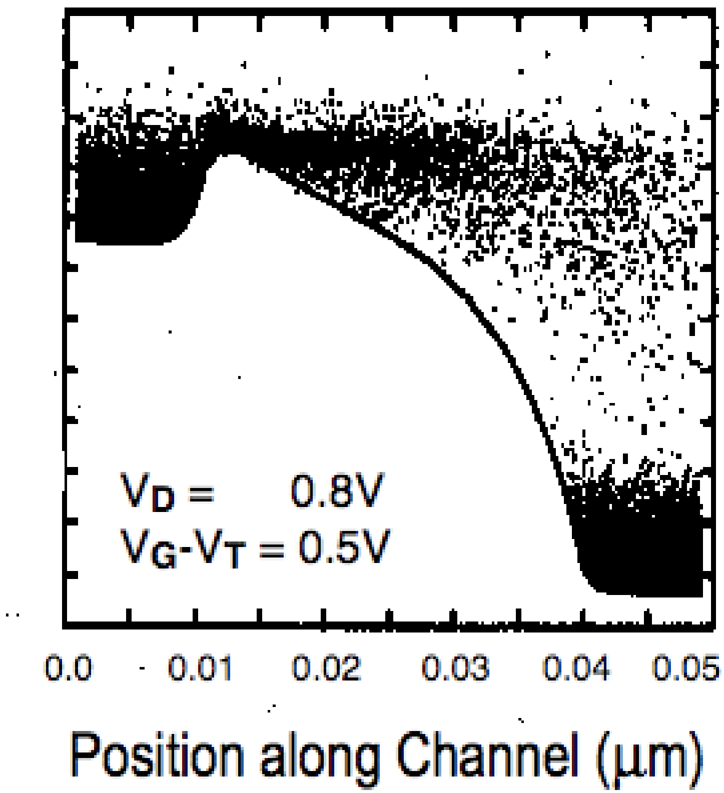


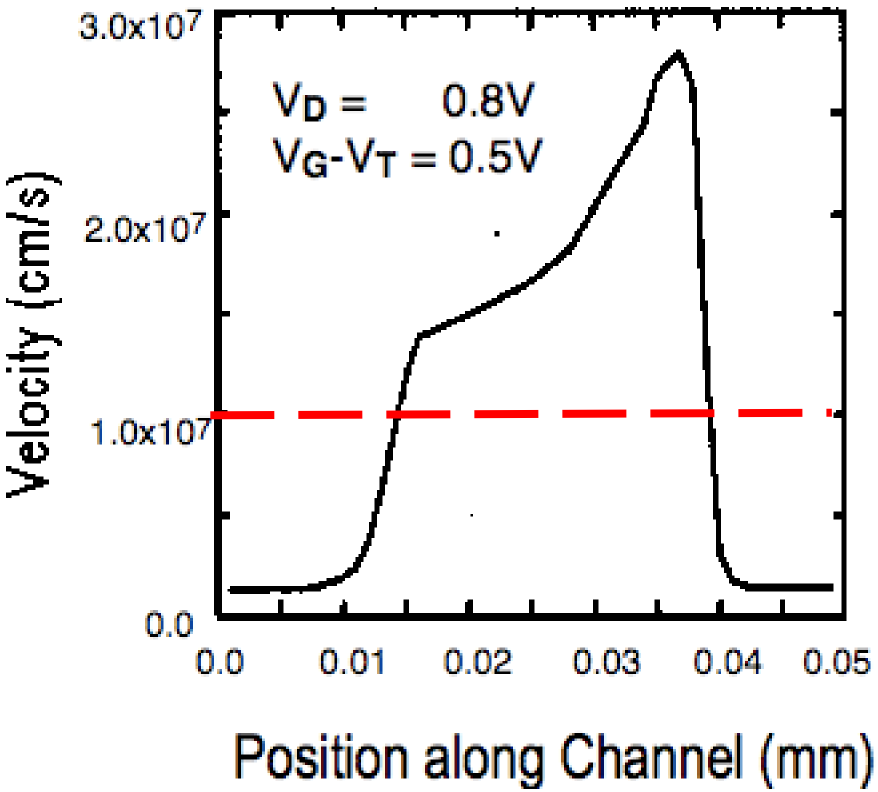
(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$I_D \propto W\left(V_{GS} - V_T\right)^{\alpha}$$

$$\alpha \approx 1$$

velocity overshoot in a MOSFET





Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

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MOSFET IV approaches

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_I(V)\,dV$$

1) "exact" (Pao-Sah or Pierret-Shields)

   see p. 117 Taur and Ning

2) Square Law $\quad Q_I(V) = -C_{ox}\left[V_G - V_T - V\right]$

3) Bulk Charge $\quad Q_I(V) = -C_{ox}\left(V_G - V_{FB} - 2\psi_B - V - \frac{\sqrt{2q\varepsilon_{Si}N_A\left(2\psi_B + V\right)}}{C_{ox}}\right)$

4) Simplified Bulk Charge $\quad Q_I(V) = -C_{ox}\left[V_G - V_T - mV\right]$

MOSFET IV approaches

5) Velocity saturation

$$Q_I(V) = -C_{ox}\left[V_G - V_T - mV)\right]$$

$$I_D = F_v \times -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_I(V)\,dV$$

$$F_v = \frac{1}{\left(1 + V_{DS}/LE_c\right)}$$

6) Full numerical

suggested reference

For a thorough treatment of MOSFET theory, see:

Yannis Tsividis, *Operation and Modeling of the MOS Transistor*, 2nd Edition, WCB McGraw-Hill, Boston, 1999.

especially Chapters 3, 4, and 6.5