Problem 1 (PDG 6.17).

We will perform the calculation for <111> silicon wafers. For <100> wafers, the linear rate constant should be divided by 1.68.

At 1000°C, in H₂O

\[
\frac{B}{A} = 1.63 \times 10^8 \exp\left(\frac{-2.05}{kT}\right) = 1.255 \mu m/ hr \\
A = 0.252 \mu m
\]

The time to grow 0.2 \mu m is then

\[
\tau = \frac{x_i^2 + Ax_i}{B} = \frac{(0.2)^2 + (0.252)(0.2)}{0.316} = 0.286 hr
\]

The time to grow the initial 0.4 \mu m was

\[
\tau = \frac{x_i^2 + Ax_i}{B} = \frac{(0.4)^2 + (0.252)(0.4)}{0.316} = 0.825 hr
\]

The extra time to grow 0.2 \mu m in the bare areas will grow some extra oxide where the initial 0.4 \mu m oxide existed

\[
x_O = \frac{A}{2} \left\{ \frac{1}{\sqrt{1 + \frac{t + \tau}{A^2 / 4B}}} - 1 \right\} = \frac{0.252}{2} \left\{ \frac{1}{\sqrt{1 + \frac{0.286 + 0.825}{(0.252)^2 / 4(0.316)}}} - 1 \right\} = 0.480 \mu m
\]

The growth of the extra 0.080 \mu m will consume 0.46 \times 0.080 = 0.037 \mu m of silicon under the thick oxide. The growth of 0.2 \mu m of oxide will consume 0.46 \times 0.2 = 0.092 \mu m of silicon. Thus, a sketch of the old and new oxide-silicon interface is shown below:

![Diagram showing the growth of oxide layers with dimensions](image)
2. (a) \( x_0^2 + Ax_0 + B(t+2) = 0 \)

Assume \( t = 0 \) \((x_i = 0)\)

At 1 atm, \( \frac{B}{A} = \frac{6.23 \times 10^6 \text{mm}^2}{1.68} \exp\left(-\frac{1.23 \text{eV}}{kT}\right) = 1.05 \times 10^{-2} \text{mm}^2/\text{h} \)

\( B \propto P_0 \), \( \frac{B}{A} \propto P_0 \), so at 0.25 atm:

\( B = 2.6 \times 10^{-3} \text{mm}^2/\text{h} \)

\( \frac{B}{A} = 1.6 \times 10^{-2} \text{mm}^2/\text{h} \)

\( x_0 = -\frac{A + \sqrt{A^2 + 4Bx}}{2} \)

\( A = \frac{B}{B/A} = 0.16 \text{mm} = 160 \text{nm} \)

\[ \begin{align*}
0.015 \text{ nm} & \text{ for } t=1 \text{h} \quad (150 \text{Å}) \\
0.028 \text{ nm} & \text{ for } t=2 \text{h} \quad (280 \text{Å})
\end{align*} \]

(b) From TCAD Software, \( x_0 = \begin{cases} 267 \text{Å} & \text{for } 1 \text{h} \\ 394 \text{Å} & \text{for } 2 \text{h} \end{cases} \)

These thicknesses are substantially larger than hand calculation due to:

1) Initial thickness of \( \sim 20 \text{Å} \) is assumed.

2) Thin oxides are assumed to grow faster than linear-parabolic model indicates as observed experimentally (Figs 6.23 and Eq. 6.37).
4. From Problem 2, FTAD software values should be the most accurate as they include thin oxide kinetics.

For first hour, oxidation rate is \( \frac{267 \AA}{1 \text{ h}} = 0.027 \text{ mm/h} \).

For second hour, \( \frac{\Delta x}{\Delta t} = \frac{394 \AA - 267 \AA}{2\text{h} - 1\text{h}} = 0.013 \text{ mm/h} \).

From Fig. 11, for \( \frac{\Delta x}{\Delta t} = \left\{ \begin{array}{l}
\frac{0.027 \text{ mm}}{\text{h}} \\
\frac{0.013 \text{ mm}}{\text{h}}
\end{array} \right\} \frac{\Delta x}{\Delta t} = \begin{array}{c}
4.7 \\
4.0
\end{array} \}
\frac{\Delta x}{\Delta t} = \begin{array}{c}
4.94 \\
4.20
\end{array}.

\( f_p = 0.95 \left( C_{s_i} = C_{s_i}^* C_V^* \right) \)

\( f_{A_i} = 0.4 \quad \frac{\Delta A_i}{\Delta A_i^*} = 0.4 \left( \frac{C_{s_i}}{C_{s_i}^*} \right) + 0.6 \left( \frac{C_V}{C_V^*} \right) = \begin{array}{c}
2.10 \\
1.82
\end{array} \)

\( f_{S_i} = 0.05 \quad \frac{\Delta S_i}{\Delta S_i^*} = 0.05 \left( \frac{C_{s_i}}{C_{s_i}^*} \right) + 0.95 \left( \frac{C_V}{C_V^*} \right) = \begin{array}{c}
0.44 \\
0.44
\end{array} \)

Note above that since \( I + V \equiv \phi \) is near equil (e.g., free)

\( C_{s_i} C_V \equiv C_{s_i}^* C_V^* \) and \( \frac{\Delta V}{\Delta V} \equiv \frac{C_{V_i}^*}{C_V^*} \).