Analyse & Design Circuits to Maximize Power Delivery

Learning goals

- Physical meaning of power
- Compute power delivered to a component
  - Given circuit diagram and component values
  - Compute power (analysis step)
- Design circuits to maximize power delivery to a component
  - Given circuit diagram
  - Determine component values

Physical meaning

- Home electricity bill
  - Bring a bill to class
- Power to run desktop vs. laptop computers
  - Which consumes more?
  - Find how much power a Pentium CPU needs
- Maximizing power: why?
  - Reduce power waste
  - Get better radio reception (more power received)
  - Make speakers sound louder
    - Find speaker input impedance as an exercise

Procedure to compute power

- Compute using general definitions
  - Valid in ALL cases
  - Instantaneous power \( p(t) \)
    - Function of time
      - \( p(t) = v(t) i(t) \)
  - Average power \( P_{\text{av}} \)
    - Average value of \( p(t) \) over a time interval \( T \) (e.g. average power use at home in one month)
    - NOT a function of time
      - \( P_{\text{av}} = \frac{1}{T} \int_0^T p(t) \, dt \)

In-class exercises

- Compute average power in components with sinusoidal signals
  - Given \( V(t) = V_m \cos(\omega t + \phi) \)
  - Average power consumed by \( R \)
  - Average power consumed by \( L \)
  - Average power consumed by \( C \)
- Observations
  - Why \( P_{\text{av}}=0 \) for \( L \) and \( C \)?
  - Faster way to compute average power for specific case of sinusoidal signals?

\( P_{\text{av}} \) consumed by \( R \)

- DC case
  - \( P = VI = V^2/R = RI^2 \)
- Sinusoidal case from previous calculation
  - \( P_{\text{av}} = V_m I_m/2 = V_m^2/(2R) = R(I_m^2/2) \)
- One ‘general formula’ for both cases?
  - Use a Root-Mean-Square value for \( v(t) \) and \( i(t) \)
  - RMS definition for any signal \( v(t) \)
    - \( V_{\text{RMS}} = \frac{1}{T} \int_0^T v^2(t) \, dt \)
In-class exercises

- Compute $V_{RMS}$ for
  - $V(t) = V_m \cos (\omega t + \phi)$
- Sinusoidal case with amplitude $V_m$ and $I_m$

\[ V_{RMS} = \frac{V_m}{\sqrt{2}} \]
\[ I_{RMS} = \frac{I_m}{\sqrt{2}} \]

Revisit $P_{av}$ by $R$

- DC case
  - $P = VI = V^2/R = RI^2$
- Sinusoidal case from previous calculation
  - $P_{av} = V_m I_m/2 = V_m^2/2R = R(I_m^2/2)$
- Use RMS value
  - $P_{av} = V_m I_m/2 = V_m^2/(2R) = (V_{RMS}^2)/R = R I_{RMS}^2$
- Same ‘general form’ using DC value and RMS value

Superposition note

- Apply 2 or more sources to a component
  - For each source, calculate $v(t)$ and $i(t)$ for the component
  - Sum all $v(t)$ to get total $V(t)$ across the component
  - Sum all $i(t)$ to get total $I(t)$ into the component
  - $P(t) = V(t) I(t)$. Calculate $P_{av}$ from $P(t)$.
  - Do NOT calculate $p(t)$ for each source and sum to get $P(t)$!!

Maximum power transfer

- Circuit with sinusoidal signals
  - Thevenin equivalent: $V_t, Z_t = R_t + jX_t$
  - Load: $Z_L = R_L + jX_L$
  - Design the load $Z_t$ to maximize power delivered to the load
    - $Z_t = Z_t^*$ or $[R_t = R_L, X_t = -X_L]$
  - Calculate maximum power for this case

Design problem

- $R = 800 \, \Omega, L = 1.6H$