Example Exam #2 Problems — EE 482  
Fall 2000

The test is open book/open notes. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses.

1. A contact is made between silicon ($\chi_s = 4.05$eV) doped with $10^{17}$cm$^{-3}$ of arsenic and aluminum ($\phi_m = 4.1$eV).

   (a) If a high density of surface states pins the Fermi level at 0.4eV above the valence band maxima, calculate $\phi_s$, $\phi_B$ and $\phi_i$ and sketch the band diagram (including vacuum level and with barriers indicated) and the charge density versus position for the contact in equilibrium. (15)

\[
\phi_s = \chi_s + \frac{kT}{q} \ln \left( \frac{N_C}{N_D} \right) = 4.05 + (0.026 \text{eV}) \ln \left( \frac{2.8 \times 10^{19} \text{cm}^{-3}}{10^{17} \text{cm}^{-3}} \right) = 4.2 \text{V} \\
\phi_B = \frac{\hbar}{q} (E_c - E_f)_{surf} = \frac{\hbar}{q} (E_g - 0.4 \text{eV}) = 1.12 - 0.4 = 0.72 \text{V} \\
\phi_i = \frac{\hbar}{q} \left[ (E_c - E_f)_{surf} - (E_c - E_f)_{bulk} \right] = 0.72 \text{ V} - 0.15 \text{V} = 0.57 \text{V}
\]

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Diagram of the band structure and charge density versus position.
(b) If a bias of 0.2 V is applied to the semiconductor relative to the metal, calculate the junction capacitance per unit area and sketch the band diagram. (10)

$$+0.2 \text{ V applied to n-type semiconductor in reverse bias}$$

For blocking contact, voltage is dropped across depletion region.

$$\Phi_i - V_a = 0.77 \text{ V}$$

Reverse bias adds to built-in voltage

$$\Phi_i = \Phi_i - V_a = 0.57 \text{ V}$$

$$= 0.77 \text{ V}$$

$$C' = \sqrt{\frac{q \varepsilon S N_D}{2(\Phi_i + 0.2V)}}$$

$$= \sqrt{\frac{1.6 \times 10^{-19} \times (11.8) \times (8.85 \times 10^{-14} \times \frac{E}{\text{um}} \times 10^{-17} \text{ cm}^{-3})}{2(0.77 \text{ V})}}$$

$$= 1.04 \times 10^{-7} \text{ F/cm}^2$$

(c) If it was possible to lower the surface state density, how would the built-in potential ($\Phi_i$) change (larger, smaller, no change)? Explain briefly. (5)

With fewer surface states, $\Phi_i$ would decrease.

The behavior of the contact would move closer to ideal behavior as lower density of surface states weakens pinning. $\Phi_{i, \text{ideal}} = \Phi_m - \Phi_S = 41 - 4.2 = -0.1 \text{ V}$

which is lower than primed value of 0.57 V.
2. In an abrupt silicon p-n junction, $N_a = 10^{18} \text{cm}^{-3}$, $N_d = 5 \times 10^{17} \text{cm}^{-3}$, $\tau_n = \tau_p = 0.25 \mu\text{s}$, $D_n = 9 \text{cm}^2/\text{s}$ and $D_p = 4 \text{cm}^2/\text{s}$ in the p-region and $D_n = 25 \text{cm}^2/\text{s}$ and $D_p = 9 \text{cm}^2/\text{s}$ in the n-region, $W_p = 0.5 \mu\text{m}$ and $W_n = 500 \mu\text{m}$. $T = 300 \text{K}$.

(a) Sketch the minority carrier densities and the hole and electron current densities as functions of position under reverse bias. Do not ignore generation/recombination in the depletion region. (8)

\[ L_{\text{np}} = \sqrt{D_{\text{np}} \tau_p} = 15 \mu\text{m} \ll W_p \]
\[ L_{\text{pn}} = \sqrt{D_{\text{pn}} \tau_p} = 15 \mu\text{m} \ll W_n \]

Actually out of scale

\[ |J_p(x_n)| < |J_n(x_p)| \]

since $W_p \ll L_{\text{pn}}$

\[ \frac{J_p(x_n)}{J_n(x_p)} = \frac{D_{\text{pn}} W_p N_a}{D_{\text{np}} L_{\text{pn}} N_d} \approx 15 \]
(b) Calculate the current density in this junction with $-2V$ applied. Assume that $x_d = x_d/4$ (8)

$$J = J_p(x_n) + J_n(-x_p) + J_{GR}$$

$$J_p(x_n) = \frac{q n_i^2 D_{pn}}{N_d L_{pn}} \left( \exp \frac{q V}{kT} - 1 \right) = -4 \times 10^{-13} \text{A/cm}^2$$

$$J_n(-x_p) = \frac{q n_i^2 D_{np}}{N_a W_p} \left( \exp \frac{q V}{kT} - 1 \right) = -6 \times 10^{-12} \text{A/cm}^2$$

$$J_{GR} = -\frac{q x_d n_i}{2 \varepsilon_0} \left( \text{under} \right) \text{reverse bias} \quad x_d' = \frac{x_d}{4} = \frac{1}{4} \sqrt{\frac{2 k_B T_0 (\phi_i - V_A) (N_a + N_d)}{q N_d N_a}}$$

$$= -1.25 \times 10^{-8} \text{A/cm}^2$$

$$= 2.7 \times 10^{-6} \text{cm} = 0.027 \mu\text{m}$$

$$J = J_p(x_n) + J_n(-x_p) + J_{GR} \equiv J_{GR} = -1.25 \times 10^{-8} \text{A/cm}^2$$

(c) If light of energy $h\nu > E_g$ was incident on this diode, causing the generation of $10^{12} \text{cm}^{-2} \text{s}^{-1}$ hole-electron pairs within a diffusion length of the depletion region, what would the new current be? (4)

$$J_{ph} = -q G = -\left( 1.6 \times 10^{-19} \text{C} \right) \left( 10^{12} \text{cm}^{-2} \text{s}^{-1} \right) = -1.6 \times 10^{-7} \text{A/cm}^2$$

$$J = J_{ph} + J_{dark} = -1.6 \times 10^{-7} \text{A/cm}^2 - 1.25 \times 10^{-8} \text{A/cm}^2$$

$$= -1.7 \times 10^{-7} \text{A/cm}^2$$
Using the diffusion approximation and assuming low-level injection and quasi-neutrality in the undepleted regions, calculate the electric field in the undepleted p region \((x < -x_p)\). Justify the diffusion approximation there \((10)\)

\[
\Delta p = \Delta n \quad (\text{quasi-neutrality})
\]

\[
= -n_{po} \frac{x}{W_p}
\]

\[
J_p = q \mu_p P E - q D_p \frac{dp}{dx} = J_i(x_n) + J_{GR} = -1.25 \times 10^{-8} \frac{A}{cm^2} \quad \text{(from (a) (16))}
\]

\[
E = \left( \frac{q D_p}{\mu_p P} \right) - \frac{1.25 \times 10^{-8} A}{cm^2} = \frac{kT}{q} \int \frac{dp}{P} - \frac{1.25 \times 10^{-8} A}{cm^2}
\]

\[
P = 10^{18}, \quad D_p = 4 cm^2/s, \quad \frac{kT}{q} = 0.026 V, \quad \frac{dp}{dx} = -\frac{n_{po}}{W_p} = -\frac{n_{po}^2}{(10^{18} \text{cm}^{-3})(5 \times 10^{-5} \text{cm})} = -4.2 \times 10^{-6} \text{ cm}^{-4}
\]

\[
E = \left( 0.026 V \right) \left( \frac{4.2 \times 10^6 \text{ cm}^{-4}}{10^{18} \text{ cm}^{-3}} \right) - \frac{1.25 \times 10^{-8} A}{cm^2} \frac{(0.026 V)}{(1.6 \times 10^{-19} \text{C})(4 \text{ cm}^2/s)(10^{18} \text{ cm}^{-3})}
\]

\[
= -1.1 \times 10^{-13} \frac{V}{cm} - 5.1 \times 10^{-10} \frac{V}{cm} = -5.1 \times 10^{-10} \frac{V}{cm}
\]

\[
|J_{n_p}^{\text{diff}}| = q \mu_p E n = q \mu_p E n_{po} \left( 1 - \frac{x}{W_p} \right)
\]

\[
= \left( 1.6 \times 10^{-14} \text{C} \right) \left( \frac{4 \text{ cm}^2/s}{0.026 \text{ V}} \right) \left( -5.1 \times 10^{-10} \frac{V}{cm} \right) \left( 2 \text{ cm}^3 \right) \left( 1 - \frac{x}{W_p} \right)
\]

\[
= 2.6 \times 10^{-24} \frac{A}{cm^2} \left( 1 - \frac{x}{W_p} \right) \ll \ll |J_{n_p}^{\text{diff}}| = 6 \times 10^{-12} \frac{A}{cm^2}
\]

Diffusion Approximation is Excellent!
A silicon p-n junction has the doping density shown below:

\[ N_d - N_a \ (\text{cm}^{-3}) \]

(a) Sketch the charge density, electric field and voltage as functions of position assuming \( x_p = 0.2 \mu m \). Determine the maximum electric field (in magnitude) and the applied bias \( (T = 300^\circ \text{K}) \) (16)

\[
\phi_i = \frac{kT}{q} \ln \left( \frac{N_d(x_0)N_a(x)N_a(x)}{n_i^2} \right) = 0.82 V
\]

\[
\phi_i - V_A = \phi_1 + \phi_2 = 4.6 V \\
(\text{just as for PIN diode})
\]

\[
\phi_2 = \frac{2q \alpha (0.1 \mu m)^3}{3 K_\infty} = 2 \left( \frac{1.6 \times 10^{-19} \text{C}}{(10^7 \text{cm}^{-3})(0.1 \mu m)^2} \right) \left( \frac{\text{cm}}{8.85 \times 10^{-14} \text{F/cm}} \right) = 1.0 V
\]

Just as in graded junction with \( x_p = 0.1 \mu m \)

\[
\phi_i = (0.1 + 0.11) = 0.22 V
\]

\[
\phi_i - V_A = 4.6 V - 0.22 V = 4.38 V
\]

\[
V_A = -V_i = -4.8 V
\]

\[
\phi_1 = \frac{q N_a (0.3 \mu m)(0.1 \mu m)}{K_\infty \infty} = 0.82 V
\]

\[
\phi_2 = \frac{2q \alpha (0.1 \mu m)^3}{3 K_\infty} = 2 \left( \frac{1.6 \times 10^{-19} \text{C}}{(10^7 \text{cm}^{-3})(0.1 \mu m)^2} \right) \left( \frac{\text{cm}}{8.85 \times 10^{-14} \text{F/cm}} \right) = 1.0 V
\]

Just as in graded junction with \( x_p = 0.1 \mu m \)

\[
\phi_1 - V_A = 4.6 V - 0.22 V = 4.38 V
\]

\[
V_A = -V_i = -4.8 V
\]

(b) At large reverse biases, by what factor will the capacitance change if the bias is increased by a factor of 2 (4)

For large reverse biases, doping is uniform \((xd >> 0.1 \mu m)\)

\[
d_{0} \propto \frac{1}{V_R^{1/2}} \quad \text{and} \quad C \propto \frac{1}{xd} \propto V_R^{-1/2}
\]

\[
V_R \times 2 \Rightarrow C \times \frac{1}{\sqrt{2}} = C \times 0.71
\]
4. In a silicon \((\chi_s = 4.15 \text{ V})\) MOS transistor with an \(n\)-type substrate and an aluminum gate \((\phi_m = 4.1 \text{ V})\), \(V_B = 0\). The substrate doping is \(N_d = 10^{16} \text{ cm}^{-3}\) and the oxide thickness is 400 Å. There is a positive oxide charge of \(Q_{ss} = 2 \times 10^{-8} \text{ C/cm}^2\) located at the oxide/semiconductor interface. There is a charge on the gate of \(Q_g = -10^{-8} \text{ C/cm}^2\).

(a) Determine the state of the channel region (accumulation, flat-band, depletion, strong inversion, etc.). (5)

\[
Q_s' = -\left( Q_g' + Q_{ss}' \right) = -\left( -10^{-8} \frac{\text{C}}{\text{cm}^2} + 2 \times 10^{-8} \frac{\text{C}}{\text{cm}^2} \right) = -10^{-8} \frac{\text{C}}{\text{cm}^2}
\]

\(Q_s' < 0\), \(n\)-type substrate \(\Rightarrow\) accumulation

(b) Sketch the charge density, electric field and energy band diagram for the system. (10)
(c) Determine the applied gate voltage. (7)

\[ V_{gb} - \phi_{ms} = V_{ox} + V_5 \]

\[ V_5 \ll V_{ox} \]

\[ \Rightarrow V_{ox} \approx \frac{Q_q}{C_{ox}} \]

\[ C_{ox} = \frac{E_{ox}}{X_{ox}} = \frac{3.9 \times 8.854 \times 10^{-14} \text{ F/cm}}{400 \times 10^{-8} \text{ cm}} = 8.63 \times 10^{-8} \text{ F/cm}^2 \]

\[ \phi_{ms} = \phi_m - \left( X_5 + \frac{(E_c - E_f)_{\text{bulk}}}{q} \right) \]

\[ = 4.1V - (4.15V + 0.21V) \]

\[ = -0.26V \]

\[ V_{gb} \approx \phi_{ms} + V_{ox} \]

\[ = -0.26V + \frac{-10^{-8} \text{ C/cm}^2}{8.63 \times 10^{-8} \text{ F/cm}^2} = -0.38V \]